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The Effectiveness of Using GeoGebra Software in Mathematics Classrooms: A Case Study of Teaching Continuous Functions in High Schools

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Abstract: GeoGebra is a math software package with several noteworthy features that make it particularly well suited for various math topics, including geometry, algebra, analysis, and statistics. The objective of this study was to assist students in promoting their activeness and initiative in learning and teaching to improve learning efficiency in mathematics with the assistance of GeoGebra software in teaching continuous functions. It was conducted with 85 students in the 11th grade, in which the students in the experimental group learned through lectures supported by GeoGebra, and the students in the control group learned through conventional teaching methods. Experimental results show that students in the experimental group with the GeoGebra-supported learning method have a more positive attitude and motivation to learn than students in the control class with the traditional teaching method. The independent t-test results of the post-test scores of the two classes also indicate that students in the experimental class have superior problem-solving skills and learning outcomes compared to students in the control class. Additionally, the findings from a student opinion survey indicate that they positively affect learning in a GeoGebra software-based learning environment.

Keywords: GeoGebra software, continuous function, learning attitudes, problem-solving skill.

新冠肺炎大流行期間數學老師在實施在線學習中遇到的困難

摘要: 地理杰布拉是一个数学软件包, 包括许多值得注意的功能, 使其特别适合用于各种数学主题, 包括几何、代数、分析和统计。本研究的目的是借助理杰布拉软件的连续函数教学, 帮助学生提高学与教的积极性和主动性, 提高数学学习效率。对 11 年级 85 名学生进行, 其中实验组学生通过 地理杰布拉支持的讲座进行学习, 对照组学生通过常规教学方法进行学习。实验结果表明, 采用地志辅助学习方法的实验组学生比采用传统教学方法的控制班学生具有更积极的学习态度和学习动机。两个班级后测成绩的独立 t 检验结果也表明, 实验班的学生与对照班的学生相比, 具有更好的解决问题的能力 and 学习成果。此外, 学生意见调查的结果表明, 它们对基于 地理杰布拉软件的学习环境中的学习产生积极影响。

关键词: 地理杰布拉 软件, 连续功能, 学习态度, 解决问题的能力。

1. Introduction

Technology is a fundamental component of modern society, and education must incorporate various aspects of technology into the teaching and learning process to be effective [1]. Many studies have shown that a technology-supported learning environment improves students' learning outcomes, problem-solving skills, and higher-order thinking [1–4]. Therefore, according

to Warner and Kaur (2017), teachers must put strategies to increase the effectiveness of their instruction [5]. To get effective teaching design when integrating technology elements, teachers must be familiar with the technical aspects of the technology, be aware of the challenges and benefits of using it to present subject matter, and choose appropriate teaching methods for their students [5].

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Students' active participation in learning and knowledge sharing in an interactive learning environment helps them acquire meaningful knowledge. As a result, a collaborative learning environment is required for mathematics education. After that, teachers' roles include presenting, explaining, and "transforming" mathematical knowledge. Many studies have shown that students' attitudes to learning mathematics, performance, and learning outcomes are directly related. Consequently, students who have a positive attitude toward learning often outperform their peers in mathematics [6]. At the same time, it is believed that when students are engaged in the lesson, they are more likely to learn effectively, and students who enjoy mathematics are more likely to put in the effort to practice and study more.

Discovery learning is a type of learning in which students build knowledge through practice and infer rules from the outcomes of those experiences, referred to as exploratory learning. Because of these characteristics, exploratory learning helps students understand knowledge to a higher degree than if the teacher's necessary information is presented only. According to several studies, discovery learning can stimulate active learning while increasing students' motivation, independence, creativity, and problem-solving skills. According to [4], cognitive resources aid students' discovery activities. According to research, GeoGebra creates an appropriate environment for students' exploration through cognitive tools, which is beneficial [4, 5].

GeoGebra is an interactive mathematical software created by Markus Hohenwater in 2002, which has features of algebraic computer systems and dynamic geometry software [5]. The fact that it combines arithmetic, geometry, algebra, and explanation in a single package means that it can be used at all levels of learning, regardless of the subject matter [3, 4, 5, 15 – 14]. The software is user-friendly, with menus and commands available in multiple languages (<https://www.geogebra.org>) [4, 7]. GeoGebra is an effective dynamic learning environment that enables users to create mathematical objects and interact differently. GeoGebra users, in particular, can create models of mathematical concepts and the relationships that exist between them. GeoGebra can test hypotheses, create realistic simulations, and simulate real-world situations when performing statistical analysis and creating statistical graphs. Additionally, this software can be obtained free via the internet. Besides, GeoGebra generates "multiple dynamically linked representations of mathematical objects" [15].

GeoGebra software is distinguished by several noteworthy characteristics that make it one of the most useful and widely used programs available. In which, the following are some characteristics that can be observed: (1) free software; (2) is open-source, i.e., it is developed in collaboration with programmers, teachers,

mathematicians, and users; (3) can run in many different operating systems; (4) can be run on many different devices; (5) strengthen the mathematical connection between geometry, algebra, analysis, statistics; (6) various representations in the form of tables, charts, equations; (7) easy to understand, friendly and easy to use; (8) can easily upload files to the website; (9) easy output to different file types; (10) Latex typing support; (11) sheet, CAS and 3D support; (12) has a harmonious look and vivid colors; (13) has a good supportive community; (14) can be used by students from high school to college; (15) regularly updated.

However, some drawbacks to using GeoGebra in the classroom should be considered, including the following: For starters, students who have never programmed before may have difficulty entering commands. In addition, some pedagogical approaches (such as independent discovery and experience) may not be appropriate for some students [4].

Because of the outstanding characteristics of GeoGebra, when combined with appropriate teaching methods, it has had a positive impact on mathematics education. Numerous studies have demonstrated that GeoGebra improves students' learning outcomes in mathematics in general [3, 7, 8, 14, 16–18]. On the other hand, numerous studies have demonstrated that GeoGebra facilitates the effective acquisition of informed and procedural knowledge, and as a result, has a positive impact on students' mathematical understanding [3, 8, 11]. Besides, studies [8, 10, 13, 16, 19, 20, 21] emphasize the positive impact of integrating GeoGebra and dynamic mathematical software in learning for students' problem-solving skills. [20]'s research created a problem-based learning environment combined with GeoGebra to train students' problem-solving skills. As a result, the problem is posed before students learn the theory in order for them to recognize the importance of solving that problem. At that time, GeoGebra was a support tool for teachers to manage students' mathematical connections and check students' problem-solving results [20]. Students' problem-solving abilities are highly correlated with their higher-order thinking abilities [21]. Studies [3, 6, 10, 14, 22] have shown the effectiveness of integrating GeoGebra and dynamic math software in general teaching mathematics for development. Higher-order thinking skills of students. [10]'s study combines a set of higher-order thinking questions with an inductive reasoning-based worksheet using GeoGebra. Results show that, on average, students' higher-order thinking skills, such as application, evaluation, and creativity, are enhanced due to this method, which is supported by research.

GeoGebra, in particular, has a significant impact on students' positive attitudes toward learning and motivation to learn and their overall perception of their learning environment as a constructive one [1, 3–5, 8, 10, 12, 14, 17, 23]. According to research [8],

GeoGebra is a supportive learning environment that can capture the attention and interest of most students while learning. In addition, students are no longer completely reliant on the teacher during the lesson and instead are encouraged to answer more questions, practice more, and thus gain a complete understanding. Moreover, students can interact with and explore concepts on their own or in collaboration with classmates. The evidence suggests that GeoGebra increases students' interest in and motivation to learn mathematics. Besides, [12] discovered through experiments that students' motivation and learning focus improved due to Geogebra. Students are placed on active learning tasks through GeoGebra, where they can discuss the tasks with their classmates and assist one another in completing them successfully. Accordingly, students show excitement in computer tasks and are curious about the results of performing certain operations. Moreover, research has shown that low-achieving students are particularly interested in GeoGebra compared to studying in regular classes. Similar to the conclusion of [8, 12] that students' roles have shifted from passive recipients of knowledge to active actors informing and reinforcing knowledge due to GeoGebra-based instruction.

On the other hand, teachers face some challenges when integrating GeoGebra into the classroom [1, 2, 4, 5, 9, 14, 18, 33]. The findings of research [1] indicate that teachers face obstacles in implementing GeoGebra in their classrooms due to a mentionable lack of training in the use of technology, a lack of necessary skills in implementing this method, a lack of confidence, an unwillingness to adopt a positive attitude toward the implementation of a new teaching method, and difficulties in integrating technology into education.

In addition to subjective reasons, there are objective factors that influence the choice and effectiveness of teachers in implementing this method, including a lack of facilities, a lack of computers, a lack of necessary resources, a lack of support and support from educational administrators, and a lack of support and support from other teachers. Schools in rural areas face additional challenges when it comes to facilities, according to a study [18].

Furthermore, according to research [14], teachers face difficulties due to a lack of time to prepare lessons involving GeoGebra. To overcome these difficulties, research [1, 2, 7, 8, 33–25] on strengthening and professional training for teachers and trainee teachers on integrating technology in general and GeoGebra in particular in teaching mathematics. The studies also recommend incorporating GeoGebra into educational curricula, investing in computer facilities and resources, and encouraging teachers to affect their subjects positively.

GeoGebra's strengths have been utilized in the teaching of a diverse range of mathematics content.

Besides studies in teaching geometry [9, 17, 18, 23], algebra [3, 9, 10, 20], arithmetic [12, 26], and even history of math [27], using GeoGebra to teach calculus has also been shown to have a positive impact [4, 5, 11, 13, 16, 28]. Research [4] and [16] have shown the effectiveness of GeoGebra in presenting a set of visual designs that help students better understand and explore basic analytic concepts such as continuity of functions at a point, to check the discontinuity of a function at a point, representing the discontinuity and the relationships between continuity and differentiability of a function of one.

The concept of a continuous function is an indispensable element in studying analytic topics such as graphs of functions, derivatives, primitives, integrals, minimum and maximum values, and equations. Because of this, increased attention and resources should be directed toward improving the effectiveness of teaching the concept of continuous functions in high school mathematics programs.

They realize the important role of continuous function for analytic contents in the program, the benefits of GeoGebra software in teaching mathematics, the relationship between discovery teachings – GeoGebra – learning attitude of students, along with the positive results of previous studies, the research team experimented with teaching the topic of continuous functions in the direction of integration with GeoGebra software.

The study's objective was to examine the impact of GeoGebra-supported teaching on student learning outcomes and problem-solving skills and their active learning attitudes, learning motivation in the lessons of continuous functions. In connection with the above research objective, the purpose of this study is to provide answers to the following questions:

1. How does the mathematics textbook teach the 11th-grade students to know about the continuous functions?
2. Can students progress significantly in their mathematical knowledge and problem-solving skills if they learn about the continuous functions while still in the phases of the learning process supported by GeoGebra software?
3. What are students' reactions when they are taught lessons using the Geogebra learning environment?

2. Research Methodology

2.1. Participants

The experiment was held at Tran Van Bay High School, Phuoc Long District, Bac Lieu Province, Vietnam. The experimental class consisted of 43 students, and the control class consisted of 42 students. Students from Tran Van Bay High School are chosen to participate in the experiment because some favorable conditions have been identified, which are as follows:

On the school side: it is possible to teach and learn with Geogebra software in the school's three information technology rooms, which are conveniently located; school leaders always facilitate research on new teaching methods (e.g., integration of GeoGebra into Math teaching) and are available to provide necessary information; colleagues are interested in incorporating GeoGebra into their classrooms, and they are actively participating in lectures and providing feedback on the research team's practical lessons.

On the student side: Students at Tran Van Bay High School are engaged and enthusiastic about the learning methods that their teachers have recommended to them.

2.2. Study Design and Instruments

The experiment was organized according to the following procedure:

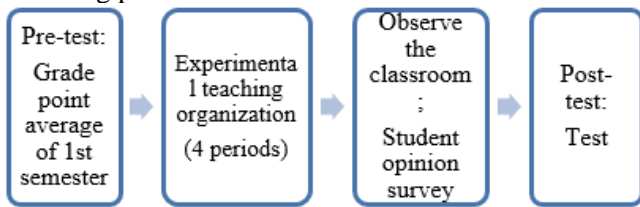


Fig. 1 Experimental progress

The study's findings resulted in teaching lessons on continuous functions tailored specifically for the experimental class. According to Fig.1, the researchers conducted class observation, post-test for the experimental and control classes. Students' opinions about the lessons were also surveyed to gauge their attitudes toward them while in the experimental class.

As previously stated, a study design consisting of a pre-test, an intervention phase, and a post-test phase was implemented to achieve the study's overall goal. In educational research, research designs such as these, which allow for the evaluation of the effectiveness of educational innovations, are extremely common, and they are used in a variety of contexts [39].

Six activities were included in the experimental teaching lesson plan, which was designed to integrate GeoGebra software into classroom instruction in order to explore concepts related to continuity of functions, and they are as follows:

Activity 1: Teaching the concept of a continuous function at a point.

Here are some examples of using GeoGebra to investigate the concept of a continuous function by observing the function graph and comparing the values of the one-sided limits at a particular point in time in the graph (Fig. 2 and Fig. 3).

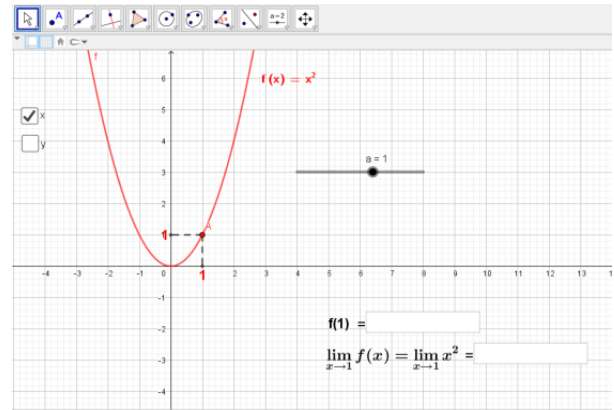


Fig. 2 Forming the concept of a continuous function at a point

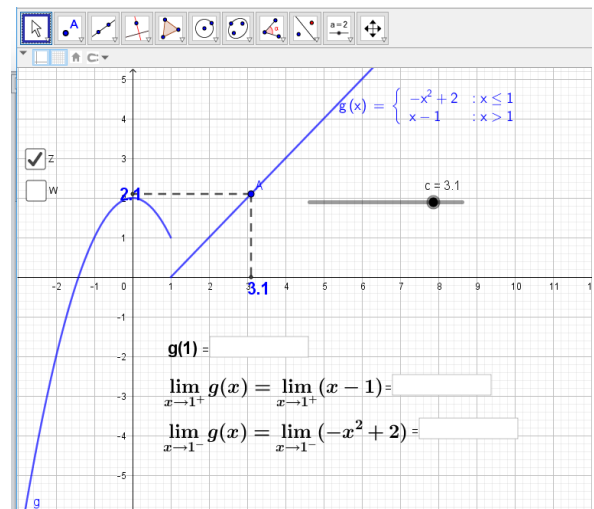


Fig. 3 Forming knowledge of discontinuous functions at a point

Activity 2: Teaching the concept of continuous functions on intervals and segments.

The following example created using GeoGebra demonstrates how to define a continuous function on an interval.

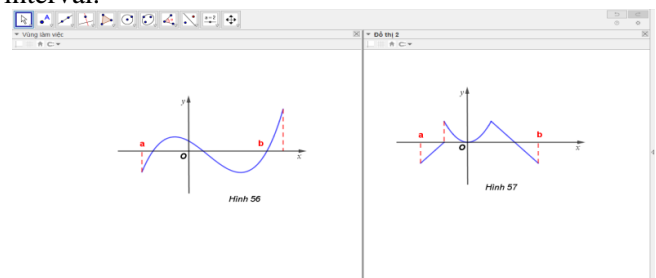


Fig. 4 Forming the concept of continuous and discontinuous functions over an interval

As shown in the image in Fig. 4, a function graph is depicted, with the teacher preparing students to simulate when a point A moves along an interval (a; b) of the graph. A solid line is formed when point A moves in the image on the left, whereas in the other image, the path of point A is broken at one point when it moves in the other image. It is the visual description of the continuity and discontinuity of a function over an interval (segment).

Activity 3: Teaching theorems about continuity of basic functions (polynomials, fractions, trigonometry).

The following example shows how GeoGebra can organize students to investigate this theorem by entering basic functions and observing their graphs, as shown in Fig. 5.

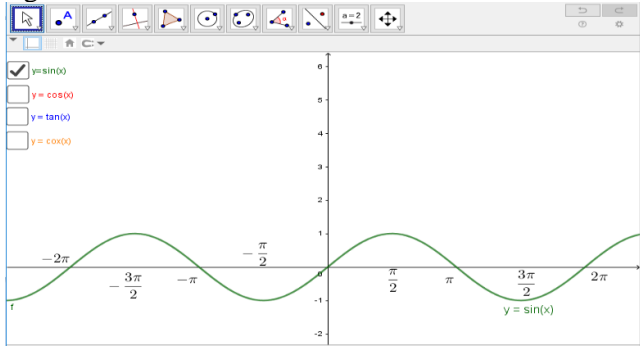


Fig. 5 Continuity of trigonometric functions

Activity 4: Teaching the theorem about the continuity of the sum, difference, product, and quotient of two continuous functions $y = f(x)$ and $y = g(x)$.

Here are some illustrations of using GeoGebra in this teaching activity. The teacher organizes students to represent the graphs of the functions $y = f(x)$ and $y = g(x)$ and explores the continuity of the sum, difference, product, and quotient functions of these two functions through their graphs (Fig. 6 and Fig. 7).

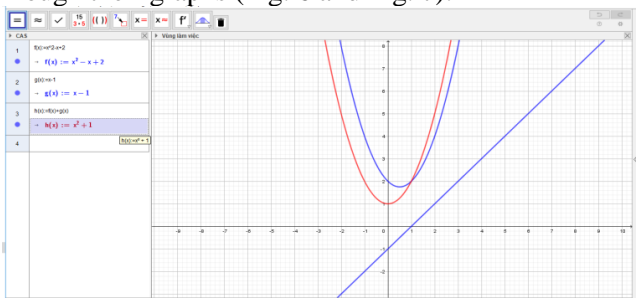


Fig. 6 The graph of a function $f(x) + g(x)$

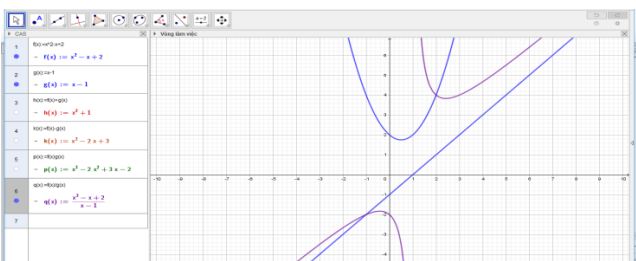


Fig. 7 The graph of a function $f(x)/g(x)$

Activity 5: Teaching the theorem about the existence of solutions on intervals (a, b) .

Here are some examples of how GeoGebra can be used in this teaching activity (Fig. 8, 9, and 10).

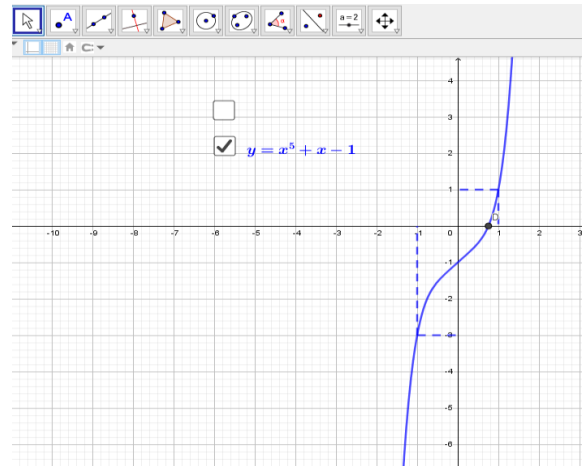


Fig. 8 The image describes the case $f(a)f(b) < 0$

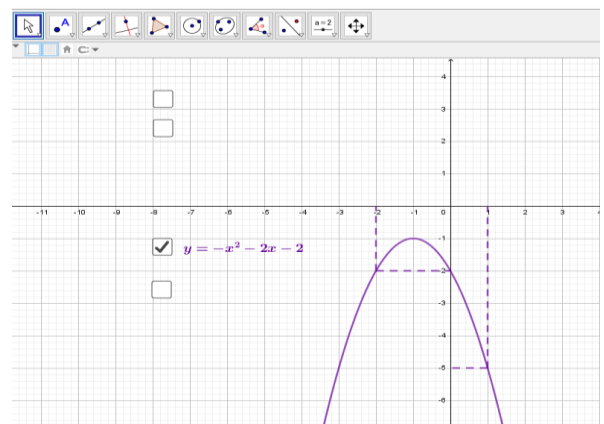


Fig. 9 The image depicts a case $f(a)f(b) > 0$

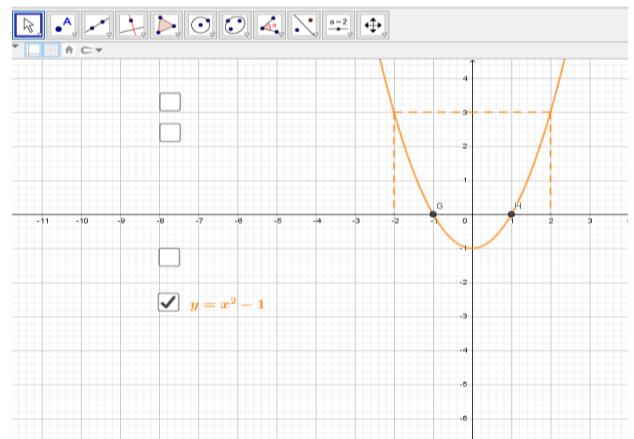


Fig. 10 The image depicts a case $f(a)f(b) > 0$

Activity 6: Teaching and solving exercises about continuous functions

With this activity, GeoGebra is used to help students (1) predict the outcome of the problem and (2) check the results of the problem after solving the problem.

These activities are integrated into the steps of the problem-solving process: (1) Understanding the content of the problem; (2) Using the dynamic features of GeoGebra software to predict the results and give directions to solve the problem; (3) Formulate a prize program; (4) Implement the solution program; (5) Check the results of the problem through the function graph designed by GeoGebra software.

Student survey questions are designed to assess students' attitudes towards functional continuum learning activities with the help of GeoGebra. The questionnaire consists of eight questions, designed based on five levels of the Likert scale, including Totally agree - Agree - Neutral - Disagree - Completely disagree.

In total, there were ten questions on the post-test, eight of which were multiple-choice and two of which were essay questions on the topic of function continuity. The content primarily tested students' understanding of and ability to apply definitions and theorems related to continuous functions in problem-solving situations.

The questions were designed to assess students' problem-solving skills in recognizing the continuity of functions based on graphs, applying properties of continuous functions at points to resolve math problems, and considering the continuity of a function at a point as well as on a set, apply the theorem about existence of solutions on an interval to prove that the equation has a solution. That was important because it contributed to confirming the content validity of the research instrument.

After conducting both post-test and pre-test experiments, preliminary research was conducted to determine whether the hypothesis was correct. Two different teachers scored the tests in two rounds, each round independent of the other. The experiment would have to be validated and tested to determine whether it would be successful. Researchers made a significant contribution to the overall effort by developing appropriate and high-quality instruments. According to two experts in mathematics education, the tests were valid and reliable, and they both agreed on this.

A high level of quality was determined to exist in the instruments and research, and several changes were implemented. They all agreed that the instrument was becoming, and an expert panel confirmed that it had not been revised further. They eventually agreed to review the tests because they believed the results were relevant to the investigation's main question.

2.3. Data Collection and Analysis

The data were collected through the mean score of the first semester of the experimental class and the control class (replacing pre-test), the results of class observations, the survey results of students' opinions in the experimental class, the test scores of the experimental class and the control class (post-test). Using the Excel and SPSS 20 software packages, the researchers conducted qualitative and quantitative data analyses.

3. Results and Discussion

Before analyzing the test scores before and after the experimental and control class experiments, the research team tested the normal distribution of the score

data with Shapiro - Wilk test through the software SPSS 20.

Table 1 Test results for the normal distribution with Shapiro – Wilk test

	Shapiro-Wilk	
	Statistic	Sig.
Pre-test of the experimental group	0.939	0.205
Pre-test of the control group	0.969	0.296
Post-test of the experimental group	0.969	0.310
Post-test of the control group	0.960	0.154

According to Table 1, test results show Sig. Values of all data points are greater than 0.05. Therefore, it can be concluded that these point data are normally distributed, which can be used in further analyses.

3.1. Pre-Test Results

Based on data from Fig. 11, it appears that the learning outcomes of the first semester of mathematics in the two classes are comparable. Whether the students' ability levels in the two classes are the same or whether they were just randomly selected. Hence, the study was conducted to test the statistical hypothesis H_0 : "There is no difference in the mean scores of the two classes" and test the above hypothesis with the independent t-test method. The results of the tests conducted using the SPSS 20 software are presented in the following table.

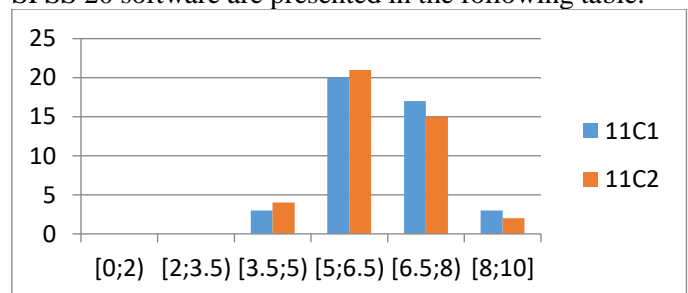


Fig. 11 Comparison chart of pre-test scores of experimental class (11C1) and control class (11C2)

Table 2 shows the independent t-test of the mean score of the first semester of the experimental and control classes with the significance level of 0.05 and the hypothesis of equal variance. Accordingly, the value sig. (2-tailed) equals 0.102 greater than 0.05, so it is acceptable to assume that the mean scores of the two classes are equivalent. Therefore, there is no difference in the math performance between the experimental and control classes.

Table 2 Results of independent t-test of the mean scores of the first semester of two classes

Group Statistics					
Group	N	Mean	Std. Deviation	Std. Error Mean	
Experimental group	43	6.6674	0.84169	0.12836	
Control group	42	6.3476	0.94126	0.14524	
Levene's Test for Equality of Variances					
F	Sig.				
0.188	0.665				
t-test for equality of means					
	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference
Equal variances assumed	1.652	83	0.102	0.31982	0.19357
Equal variances not assumed	1.650	81.514	0.103	0.31982	0.19383

3.2. Lesson Evaluation Results

After conducting an extensive series of experiments, the research team discovered that using GeoGebra software to support the teaching of continuous functions was consistent with reality and followed the lesson's content.

Primarily, students became accustomed to using GeoGebra software to discover new information. Accordingly, students were able to approach concepts and theorems in visual geometry and observe many different cases of the same object of knowledge statue thanks to the intuitive and dynamic images provided by GeoGebra software. As a result, cases that could have confused students' understanding were successfully addressed during the lesson. Second, students engaged in the development of lessons in a highly active manner, enthusiastically participating in the activities assigned by the teacher. This activity contributed to the lively atmosphere that existed in the classroom. Third, in general, after the lesson, students understood the content of the concepts and theorems that they had learned, considered the continuity of the function, and were able to apply the knowledge they had gained to cope with specific exercises given them. The results presented above demonstrate that Geogebra effectively stimulated active learning and improved students' understanding of the lesson.

Besides, students were trained in components of problem detection and solving capabilities such as information gathering, data processing, determining how to address problems, evaluating solutions to find optimal solutions, calculating and applying them in practice based on designing compatible educational activities during lessons with the assistance of the GeoGebra environment.

3.3. Results Related to Post-Test

3.3.1. Quantitative Analysis

On a 10-point scale, here is a list of the test results for the control class and the experimental class, which are both presented in the following section:

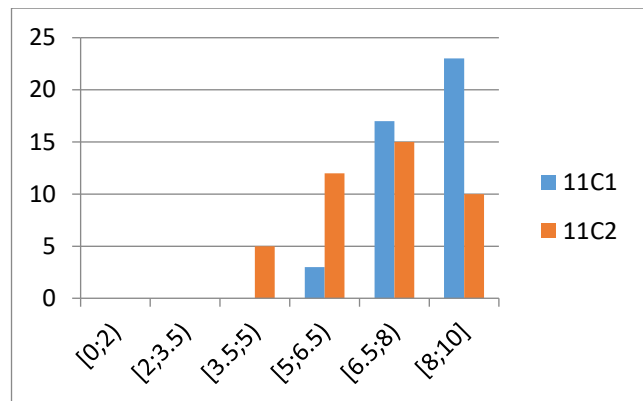


Fig. 12 Graph of post-experiment test scores of experimental class (11C1) and control class (11C2)

Statistical results in Fig. 12 show that good scores in the experimental class were very high, with 23 assignments with scores in the range [8, 10]. Meanwhile, the control class with this grade encountered only ten problems. The number of assignments with good scores [6.5; 8) of the experimental class was higher than the control class, but the difference was not too large. In particular, there were no assignments with scores below the average (lower than 5 points), while the control class had five assignments with scores in the range [3.5; 5). Additionally, the researchers used the SPSS 20 software to conduct an independent t-test on two data points to determine whether there was a difference between the post-test results of the two classes.

Table 3 Results of independent t-test of the mean scores of the post-test of two classes

Group Statistics					
Group	N	Mean	Std. Deviation	Std. Error Mean	
Experimental group	43	7.9302	1.10508	0.16852	
Control group	42	6.5714	1.38622	0.21390	
Levene's Test for Equality of Variances					
F	Sig.				
3.500	0.065				
t-test for equality of means					
	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference
Equal variances assumed	5.003	83	0.000	1.35880	0.27159
Equal variances not assumed	4.990	78.261	0.000	1.35880	0.27231

Table 3 shows the results of an independent t-test of the mean post-test scores of the experimental and control classes, with a significance level of 0.05 and the hypothesis of equal variance. Accordingly, the value sig. (2-tailed) was equal to 0.00 less than 0.05, verifying that the difference between the two mean values of post-test scores of the two classes was statistically significant. Aside from that, it can be concluded that the

experimental class's post-test results were significantly better than those of the control class were.

The degree of influence (ES), according to Cohen, indicates the magnitude of the pedagogical effect, which was expressed through the standard mean difference (SMD). The calculation results show that the SMD value is 0.98. Accordingly, the degree of ES influence was relatively large. As a result, the pedagogical experiment had a disproportionately large impact on the post-test results of the experimental class when compared to those of the comparison class.

3.3.2. Qualitative Analysis

According to the post-test analysis of their work, students in the experimental class demonstrated superior problem-solving abilities than students in the control class. Specific to the multiple-choice questions, most students in the experimental class received a perfect score, whereas no student in the control class received a perfect score in this section. For essay questions involving considering the continuity of a function at a point, the majority of students' work in the experimental class demonstrates that they could determine the requirements of a problem, grasp and capture the information of a problem well, master the process of considering the continuity of a function at a point, present a solution clearly and completely, and formulate a solution that was clear and complete. Since the beginning of the control class, many exercises had contained errors in determining the elements of problems and the direction of problem-solving, resulting in interruptions or failures to provide a solution. Some problems did not ensure the correct procedure for considering the continuity of functions, some problems did not have logic, and some problems did not have logic due to a misunderstanding of the logic of the exercises. For the question in which the students were required to use the intermediate value theorem to prove a solution to a continuous function, many exercises in the experimental class demonstrated that they had correctly identified the proof and constructed reasonable arguments.

Students gained new knowledge and learned new methods of acquiring that knowledge while participating in GeoGebra-based instruction. They also developed positive and creative thinking skills and were better prepared to adapt to social situations, detect and resolve problems as they arose quickly and reasonably. Instead, students were taught to discover and deal with problems, which was a beneficial way to train them to discover and handle problems in the future.

Meanwhile, many students in the control class could not provide a solution or provided an incorrect solution. This result was observed because they had not correctly identified the math or had not mastered solving it. In conclusion, students in the experimental class demonstrated superior problem-solving abilities in mathematical forms of continuous functions than those

in the control class, as evidenced by their superior performance.

3.3. Student Opinion Survey Results

In order to assess more specifically the learning effectiveness and activeness of students in the lessons, after teaching the experiment, a survey was conducted to collect students' opinions in the experimental class. The survey results are listed in the following tables.

Table 4 Results of student feedback on Item 1

Item 1: When learning the definition of a continuous function at a point with the help of GeoGebra software, I can quickly discover the concept					
Levels	Totally agree	Agree	Neutral	Disagree	Completely disagree
%	44.2%	46.5%	9.3%	0%	0%

Based on data from Table 4, in Item 1, when forming the concept of a continuous function at a point, the number of students who said they could detect the concept quickly accounted for 90.7%, and no students said they disagreed with the effect of the figure concept with the help of GeoGebra. For the first time, the experimental class had access to a teaching method in which definitions were built through visual simulations using GeoGebra software, and this was the first time that happened. As a result, students may still be perplexed and unable to adapt to the software right away. The results of the survey, on the other hand, show that the method of incorporating GeoGebra into the classroom had positive effects.

Table 5 Results of student feedback on Item 2

Item 2: When learning the definition of a continuous function on an interval, I can quickly discover the concept with the help of GeoGebra software					
Levels	Totally agree	Agree	Neutral	Disagree	Completely disagree
%	65.1%	30.2%	4.7%	0%	0%

According to Table 5, for Item 2, when asking students' opinions about forming the concept of a continuous function on an interval (segment) with the help of GeoGebra software, the majority of students agreed that they could quickly discover knowledge (accounting for 95.3%), and no students stated that they learned this knowledge slowly or that they did not learn this knowledge at all. In teaching design, after showing the graph of continuous/discontinuous functions on an interval, the teacher used GeoGebra's "dynamic" to illustrate the continuity/discontinuity of the function on the interval. Consequently, it can be concluded that this instructional design was effective in conceptual formation among students.

Table 6 Results of student feedback on Item 3

Item 3: When learning theorems about continuous functions with the help of GeoGebra software, I can discover theorems quickly					
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Levels	Totally agree	Agree	Neutral	Disagree	Completely disagree
%	79.1%	20.9%	0%	0%	0%

In particular, the results of Item 3 in Table 6 demonstrate that when forming theorems on the continuity of functions with the aid of GeoGebra software, most students discovered that they were able to discover knowledge, with 79.1% strongly agreed quickly, and 20.9% agreed. Teachers could help students understand the theorem more deeply and comprehensively by providing vivid, specific, and time-efficient illustrations of specific cases of the theorem. While applying the theorem to their situations, students would not be interrupted in any way due to this. Furthermore, to use GeoGebra software to illustrate different representations of concepts, such as during concept formation activities, GeoGebra software was found to positively affect students when teaching theorems.

Table 7 Results of student feedback on Item 4

Item 4: When I study with GeoGebra's support, I can master the knowledge better

Levels	Totally agree	Agree	Neutral	Disagree	Completely disagree
%	58.1%	41.9%	0%	0%	0%

Additionally, the survey results for Item 4 in Table 7 show that the majority of students' responses to their mastery of definitions and theorems were very good, indicating that they had learned new concepts and properties of continuous functions as well. Indeed, 58.1% of students completely agreed, and 41.9% agreed with the statement. The results of this question were consistent with students' responses to items 1, 2, and 3. Thus, students' successful discovery of concepts and theorems contributed to their deeper understanding, resulting in a long-term knowledge occupation. Correspondingly, students would have a better grasp of the knowledge.

Table 8 Results of student feedback on Item 5

Item 5: After the lesson on continuous functions with the help of GeoGebra, I can understand the process of considering the continuity of functions better

Levels	Totally agree	Agree	Neutral	Disagree	Completely disagree
%	51.2%	48.8%	0%	0%	0%

This lesson on continuous functions had several important objectives: to ensure that students understood the continuity of functions into account. In Item 5, when asked if studying with GeoGebra helped students understand functional continuity better, most students responded that they could master the process, with 51.2% of students completely agreeing and 48.8% of students agreeing with the statement (Table 8).

Table 9 Results of student feedback on Item 6

Item 6: In the lesson solving exercises with the support of

GeoGebra software, I have a higher interest in learning					
Levels	Totally agree	Agree	Neutral	Disagree	Completely disagree
%	62.8%	37.2%	0%	0%	0%

Item 6 was designed to gauge students' interest in learning while they were engaged in physical activity with the assistance of GeoGebra. Through dynamic software, students were able to predict and check the results of the problem for the first time during a problem-solving period, which was a first in the problem-solving period. The result in Table 9 was that 62.8% of students expressed a strong interest in the lesson, and 37.2% expressed an interesting attitude. This result also partially reflected the impact of using GeoGebra to predict and check problem results on students' learning motivation formation, which was a significant finding. Using previous problem results as a guide, students could overcome the problems with greater clarity and make greater efforts to verify their predictions.

Table 10 Results of student feedback on Item 7

Item 7: I love these practical lessons with the help of GeoGebra software

Levels	Totally agree	Agree	Neutral	Disagree	Completely disagree
%	81.4%	18.6%	0%	0%	0%

Students' attitudes toward continuous function topics were assessed using GeoGebra software in Item 7, which was given to them to see how well they understood them. Therefore, as shown in Table 10, 100% of students were satisfied with these lessons, with 81.4% completely agreeing. Adapting to an ever-changing society where technology played an important role was something that most students were doing. Meanwhile, the GeoGebra software was a product of technological advancement. Consequently, the fact that teachers created a learning environment that included GeoGebra corresponds to the interests and development trends of the vast majority of students in the classroom. Furthermore, the high level of interactivity in the GeoGebra learning environment piqued students' interest in mathematics. As a result, student feedback regarding their interest in GeoGebra-based lessons was completely legitimate.

Table 11 Results of student feedback on Item 8

Item 8: I want to study with the help of GeoGebra software in other lessons

Levels	Totally agree	Agree	Neutral	Disagree	Completely disagree
%	88.4%	11.6%	0%	0%	0%

When asked about their desire to use GeoGebra in other lessons in Item 8, according to Table 11, all students responded in agreement (11.6%) or strongly agreed (88.4%). When compared to the responses to previous questions, the results of this question were consistent with the results of previous questions

regarding students' active learning attitude and knowledge discovery efficiency. Accordingly, the research team believes that the integration of GeoGebra in teaching should be considered in three aspects: frequency, duration, and variety of math topics covered.

Besides, the student survey results indicate that students had a positive attitude toward incorporating GeoGebra into their math classes. In response to questions about the effectiveness of knowledge discovery, students' responses provided sufficient evidence for the researchers to conclude that the formation of concepts and theorems for students through GeoGebra with appropriate designs could aid the students to form and understand knowledge more quickly, comprehensively, and deeply. Additionally, the students' survey results in the experimental class were unanimous in their belief that their opinions corresponded to the results observed in the lessons regarding the students' level of interest and participation in the lessons. Thereby, it shows that the integration of GeoGebra in the lessons positively affected students' learning attitudes and the classroom atmosphere and could thus improve teachers' teaching effectiveness. This result also served as a basis for teachers to consider implementing this teaching method to contribute to mathematics education innovation while increasing their students' activeness and initiative.

4. Conclusion

The experiment results show that the continuous teaching of functions with the assistance of GeoGebra software positively affects students' attitudes, motivation for learning, and learning outcomes in mathematics. According to previous research on incorporating GeoGebra into the teaching of continuous functions, this result is accurate [4, 16]. Learning environments that incorporate technology-assisted instruction are considered appropriate for many students during this period of scientific and technological advancement 4.0. Learning through GeoGebra software can, primarily, draw the attention of the majority of students to the software itself, stimulate creativity, and increase interaction between teachers and students and between students and other students through the exchange of ideas, among other benefits. GeoGebra can be used in conjunction with appropriate teaching methods to create a learning environment that is active, creative, and highly interactive for students. Studies [1, 3–5, 8, 10, 12, 14, 17, 23] have also revealed relevant outcomes. Because of classroom observations and student surveys, have discovered that students actively participate in knowledge formation activities, predict, and check problems' outcomes when teaching continuous functions using GeoGebra software.

For this reason, students will explore their knowledge and put their predictions to the test when implementing this new learning method. It is possible to pique students' interest in learning, and they are

encouraged to investigate and research related knowledge. In contrast, the knowledge-forming effects of GeoGebra teaching contribute significantly to students' understanding of the lesson and, as a result, to improved student learning outcomes in the end. Research [1, 3–5, 8, 10, 12, 14, 17, 22, 23] pointed out the close relationship between learning attitudes and students' learning outcomes about the learning environment with information technology application and especially GeoGebra.

GeoGebra is a mathematical software that combines geometry, algebra, and calculus with a dynamic drawing system that makes it easy for users to perform logical mathematical representations. In comparison to many other software packages, this is a significant advantage. Aside from that, GeoGebra software assists teachers in creating teaching situations for geometry concepts, properties, and theorems in an intuitive and exploratory manner. On the other hand, GeoGebra can promote active and student-centered learning by enabling mathematical experiments, interactive exploration, and learning discovery.

Although each student has a unique learning capacity, each class has a unique learning capacity as well. The audience receiving the information and the designer's ability to combine the information determines the location of GeoGebra software in the lesson content and its presentation.

Aside from the fact that the findings of this study are reliable, it also has some limitations. The research was conducted with 85 grade 11 students throughout four lessons. The sample size is not particularly large, and the fact that the experimental time is relatively brief should not be overlooked. Although the experimental results indicate that the student's learning attitude is extremely positive and the test results indicate that the pedagogical impact has had an impact, the long-term learning motivation and the specific learning effect on the skills cannot be determined within the scope of the study because of the limitations of the research design.

Future research will examine the impact of incorporating GeoGebra into classroom instruction on developing long-term learning motivation in students and good math skills. It is possible to consider general abilities in order to provide clarification. There are various approaches to overcoming difficulties in teaching GeoGebra, including technical aspects of teachers' and students' teaching design methods, teaching met, and student assessment standards. For its part, whether it is done using GeoGebra specifically or using technological means in general, mathematics education must be compatible with the learning objectives and content of each country's educational program. As a result, the proposal of [30] on investigating the design, development, and implementation of mathematics education programs in Vietnam in the context of international cooperation, technological development, and application is being

considered. Furthermore, information and communication technology and educational innovation in the development of learners' competencies require the attention of domestic mathematics education researchers.

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