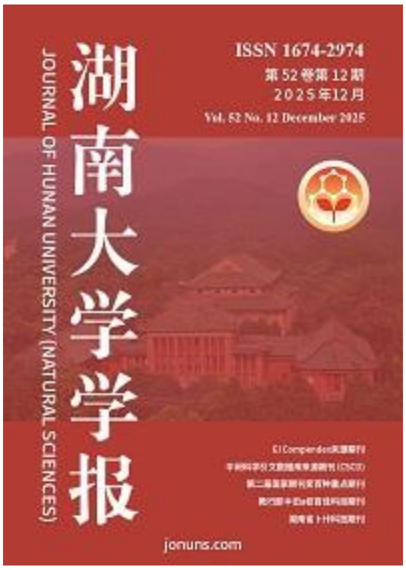

Journal of Hunan University

(Natural Sciences)

Vol. 52 No. 12

December 2025

Available online at

https://jonuns.com

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Open Access Article https://doi.org/10.55463/issn.1674-2974.52.12.9

Strategic Fading of Scaffolding to Foster Mathematical Autonomy: Supporting the Shift from Descriptive to Symbolic Thinking in Elementary Proportional Reasoning

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Article history

Received: November 20, 2025

Revised: December 22, 2025

Accepted: January 6, 2026

Published: January 30, 2026

Abstract: Elementary students' mathematical thinking is frequently constrained by persistent misconceptions and an overreliance on procedural instruction. International assessments of mathematical literacy consistently report lower levels of achievement among students in many developing countries, underscoring the need for instructional approaches that promote conceptual understanding rather than rote learning. Scaffolding, understood as temporary and adaptive instructional support, has been widely acknowledged as an effective means of facilitating students' conceptual development. Nevertheless, its classroom enactment—particularly the processes through which support is responsively adjusted and gradually withdrawn—remains insufficiently documented and systematically analyzed in empirical research.

This study aims to examine the forms of scaffolding employed by teachers, their responsive strategies in addressing student errors, and the observable indicators of scaffolding reduction (fading) in mathematics instruction grounded in visual pattern recognition and comparative reasoning. A descriptive qualitative methodology was adopted, using a case study design involving three upper elementary school students. Data were collected through analyses of students' written work, classroom interaction observations, and semi-structured interviews. The data were analyzed thematically within the framework of contingent scaffolding.

The findings indicate differentiated learning trajectories among the participants. Student MA demonstrated a shift from intuitive verbal descriptions to symbolic comparative reasoning following interactive scaffolding. Student RFM exhibited independent formal reasoning from the outset, requiring minimal instructional support. In contrast, student IAM experienced a substantial conceptual transition after receiving explicit instructional intervention. Notably, all three students were ultimately able to generalize that the number of blue triangles was consistently less than half of the total number of triangles.

These results highlight the critical role of adaptive and contingent scaffolding in fostering conceptual understanding and learning autonomy in elementary mathematics. By documenting the forms, timing, and transitions of instructional support, this study contributes to the empirical literature on scaffolding practices in primary education. Importantly, the findings provide a novel account of how scaffolding is dynamically enacted and strategically faded in response to students' errors, enabling a progression from descriptive to symbolic proportional reasoning. The identification of concrete indicators of scaffolding reduction aligned with students' emerging autonomy offers theoretically grounded and practice-oriented implications for the design of adaptive instructional support in elementary mathematics classrooms.

Keywords: instructional scaffolding; mathematical reasoning; mathematics education; proportional reasoning; gradual fading of support.

通过策略性渐隐支架促进数学自主性：支持小学比例推理中从描述性思维向符号性思维的转变

摘要：小学生的数学思维常常受到固有误解和过度程序化教学的限制。国际数学素养评估显示，许多发展中国家的学生成就水平持续偏低，这凸显了促进概念理解而非死记硬背的教学方法的必要性。支架教学（scaffolding）作为一种临时且适应性强的教学支持，被广泛认为是促进学生概念发展的一种有效策略。然而，在课堂中支架的实施—尤其是如何根据学生的错误进行响应性调整并逐步撤销支持的过程—在实证研究中仍缺乏系统记录和分析。

本研究旨在探讨教师使用的支架形式、应对学生错误的响应策略，以及在基于视觉模式识别和比较推理的数学教学中可观察到的支架减少（fading）指标。研究采用描述性定性方法，并以个案研究设计为基础，涉及三名小学高年级学生。数据通过学生书面作业、课堂互动观察以及半结构式访谈收集，并在应变支架（contingent scaffolding）框架下进行主题分析。

研究结果显示，参与学生的学习轨迹各不相同。学生 MA 在互动支架的帮助下，从直观的口头描述转向符号化的比较推理；学生 RFM 从一开始就表现出独立的正式推理能力，几乎不需要教学支持；而学生 IAM 在接受明确的教学干预后经历了显著的概念跃迁。值得注意的是，三名学生最终都能够概括出蓝色三角形的数量始终少于总三角形数量的一半。

这些结果强调了适应性和应变支架在促进小学数学概念理解与学习自主性方面的关键作用。通过记录支架的形式、时机及转变过程，本研究为小学教育中支架实践的实证文献提供了贡献。尤其重要的是，本研究提供了支架如何根据学生错误动态施行并策略性撤销的实证案例，从而支持学生从描述性推理向符号化比例推理的转变。通过识别与学生自主性发展相一致的具体支架减少指标，研究为小学数学课堂中设计适应性教学支持提供了理论与实践指导。

关键词：教学支架；数学推理；数学教育；比例推理；支架逐步撤销

1. Introduction

Mathematical competence is a critical foundation for addressing the global challenges of the 21st century, particularly in the domains of critical thinking, problem-solving, and data-informed decision-making. However, mathematical literacy levels among students in many countries—especially developing nations—remain concerning. The 2018 PISA report revealed that approximately 76% of 15-year-old students in developing countries had not yet reached the minimum proficiency level in mathematical literacy, reflecting deficiencies in mathematical reasoning and thinking skills [1]. This condition indicates that mathematics instruction often continues to emphasize procedural routines and final answers, rather than fostering students' cognitive processes and learning autonomy.

One pedagogical approach shown to be effective in enhancing students' mathematical thinking is scaffolding, which refers to temporary support provided by teachers during learning to help students accomplish goals they cannot yet achieve independently (van de Pol, Volman, & Beishuizen, 2010). Scaffolding enables progressive learning tailored to students' developmental levels. This approach not only helps students grasp complex concepts but also addresses the misconceptions that frequently hinder learning [2].

Anghileri [3] categorized scaffolding into three levels—explicit, interactive, and reflective—all of which play vital roles in guiding students from procedural mastery toward deeper conceptual understanding. In practice, teachers are expected to deliver support contingently, adjusting the type and intensity of scaffolding based on students' needs and responses during instruction [4]. Scaffolding can also be directed metacognitively by prompting students to reflect on their own thinking processes or teach concepts back to virtual agents, which has been shown to improve algebraic understanding [5].

Technological advancements have expanded the forms of digital scaffolding. Simulation-based learning and flipped classrooms supported by Learning Management Systems (LMS) equipped with scaffolding tools have enabled pre-service mathematics teachers to develop a deeper understanding of geometry [6]. Scaffolding can also be implemented through visual representations and multi-modal conceptual models in gamified learning environments, which enhance students' motivation and engagement [7]. Embodied learning and concreteness fading approaches are also employed to assist learners in transitioning from concrete to abstract thinking, proving effective in improving STEM learning [8].

Scaffolding has been particularly important in supporting students with special needs. English learners (ELs) and high-risk students benefit from visual, linguistic, and conceptual scaffolds that have significantly improved their mathematical problem-solving skills [9]. In addition, using learning trajectories as a structural form of scaffolding helps teachers understand students' developmental progressions and design instruction aligned with cognitive stages [10].

To avoid fostering dependence, scaffolding must be gradually withdrawn through a process known as fading. Support is reduced as students begin to demonstrate independence in completing tasks, understanding concepts, and applying problem-solving strategies [11], [12]. Data-driven adaptive technologies now allow for real-time adjustments to scaffold intensity based on students' individual needs [13]. Even in game-based learning environments, adaptive scaffolding has been shown to enhance engagement and learning outcomes without compromising learner autonomy [14].

Although many researches have demonstrated the effectiveness of scaffolding, there remains a lack of comprehensive documentation of its practical implementation, especially in mathematics education. This research aims to fill that gap by describing the forms of scaffolding, the steps taken to provide support when students make errors, and the indicators used for systematically fading assistance. By deepening our understanding of these practices, teachers will be better equipped to apply scaffolding effectively to foster student autonomy, correct misconceptions, and strengthen mathematical understanding.

2. Literature Review

2.1. Types and Strategies of Scaffolding

Various forms of scaffolding have been identified in recent educational research. Metacognitive scaffolding, such as teaching students to reflect on their thinking processes or to re-explain concepts to peers, has proven effective in enhancing mathematical problem-solving abilities [5]. Cognitive scaffolding, which includes direct feedback and step-by-step prompts, has been applied in simulation-based learning environments; however, its effectiveness is highly context-dependent [13].

Visual-based approaches and conceptual models have also shown promising outcomes. For instance, multi-representational scaffolding in online gamified learning environments has been found to improve students' motivation and engagement [7]. The concreteness fading strategy, which transitions from concrete to abstract representations, has been reported to strengthen students' spatial abilities and conceptual understanding in STEM subjects [8].

For students with special needs, such as English Learners, linguistic and visual scaffolding approaches are particularly beneficial. Conceptual model-based programs supplemented with visual supports have been demonstrated to improve the problem-solving performance of at-risk students [9]. Additionally, scaffolding is increasingly utilized in teacher education, such as through close reading approaches to support understanding of mathematical texts [15], and through the use of learning trajectories to trace the development of students' mathematical understanding [10].

2.2. Scaffolding in Response to Student Errors

When students make errors, scaffolding can serve as both a diagnostic tool and an intervention mechanism. Common scaffolding stages in such contexts include: (1) identifying misconceptions, (2) providing contingent support, (3) using failure as a pedagogical tool (failure-driven scaffolding), (4) gradually fading assistance, and (5) prompting reflection on errors [13], [16], [17]. This type of support must be flexible and responsive to the individual needs of students, whether in small group instruction or whole-class settings [18]

2.3. Fading of Scaffolding

As students gain greater independence, scaffolding should be gradually reduced. Criteria for fading support include students' ability to complete tasks independently, active participation in discussions, and the demonstration of adequate conceptual understanding [11], [12]. Adaptive technologies such as learning management systems (LMS) and simulation-based environments also facilitate this process by automatically adjusting the level of support in real-time based on students' performance [13], [19].

In digital game-based learning environments, adaptive scaffolding not only enhances students' performance and self-regulation but also strengthens affective dimensions such as interest and motivation [14], [20]. Moreover, emotionally responsive scaffolding design has been shown to influence students' readiness to face new challenges independently [21].

3. The Research Methods

3.1. Research Approach and Design

This research employs a descriptive qualitative approach with a case study design to deeply explore the scaffolding practices employed by mathematics teachers in responding to students' errors during classroom instruction. This approach was selected for its capacity to explain educational phenomena contextually and holistically, particularly in identifying the forms, stages, and dynamics of scaffolding reduction during classroom interactions. The case study design enables the researcher to directly observe the teacher's role in facilitating students' cognitive development, including how support is provided and gradually withdrawn in accordance with students' zones of proximal development [22], [23].

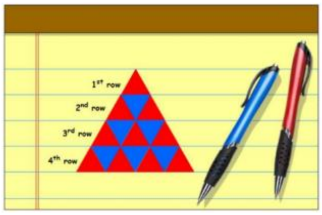
3.2. Participants

The participants in this study were three upper elementary school students from Hasyim As'ari Elementary School in Malang City. They were purposively selected to capture variations in students' initial mathematical understanding, particularly with respect to visual patterns and proportional reasoning, as well as their responsiveness to instructional scaffolding.

The selection criteria included: (1) differences in prior understanding of visual patterns and proportional reasoning, as identified through preliminary classroom observations and teacher recommendations; (2) students' ability to articulate mathematical reasoning both verbally and in written form; and (3) their level of engagement during mathematics instruction.

3.3. Instrument

The instrument used in this study was adapted from items of the *Programme for International Student Assessment (PISA)* 2022 [1]. The instrument was designed to elicit students' mathematical thinking, particularly in the context of visual pattern recognition and proportional reasoning.



Source: Adapted from PISA 2022

"Alex created a pattern of red and blue triangles. He stated that if he continues to add more rows to the pattern, the number of blue triangles will always be less than half.

Do you agree with Alex? Explain your reasoning!"

Figure 1. Instrument task

3.4. Research Instrument Data Collection Techniques

Data were collected through three main methods: (1) students' written work on visual pattern tasks, (2) classroom observation focusing on teacher-student interactions, and (3) unstructured interviews with both the teacher and the students. Method triangulation was employed to ensure the validity and trustworthiness of the findings [24].

3.5. Research Procedures

The data collection procedure consisted of the following steps: Task administration, involving a visual pattern problem featuring geometric objects (red and blue triangles); Documentation of students' problem-solving processes and the corresponding teacher interventions; Recording and analyzing the interactions between the teacher and students to identify forms and transitions of scaffolding support.

Scaffolding was categorized into three types based on frameworks from Anghileri [3] and Smit, van Eerde, & Bakker [25]: Explicit scaffolding: direct instructional support; Interactive scaffolding: open-ended questioning aimed at guiding student reasoning; Reflective scaffolding: prompts designed to stimulate students' conceptual generalization.

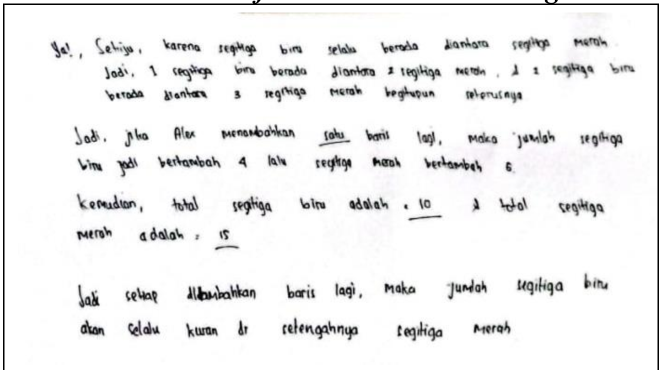
3.6. Data Analysis

Thematic analysis was employed using the contingent scaffolding framework, where teacher support is dynamically adjusted based on students' responses and abilities [4]. The process of fading—the gradual withdrawal of assistance—was traced using the following indicators: Students' consistent performance in solving problems independently; The use of formal representations, such as fractions and comparative symbols; and The transfer of problem-solving responsibility from the teacher to the student [26], [27].

Each unit of analysis—including transcribed teacher-student interactions and students' written responses—was repeatedly examined to identify shifts in the level of scaffolding and progress in students' mathematical thinking.

4. Results and Discussion

4.1. MA: *Guided Shift to Fractional Thinking*


Ya!, Setuju, karena segitiga biru selalu berada diantara segitiga merah. Jadi, 1 segitiga biru berada diantara 2 segitiga merah, 2 segitiga biru berada diantara 3 segitiga merah begitupun seterusnya

Jadi, jika Alex menambahkan satu baris lagi, maka jumlah segitiga biru jadi bertambah 4 lalu segitiga merah bertambah 6.

Kemudian, total segitiga biru adalah = 10 & total segitiga merah adalah = 15

Jadi setiap ditambahkan baris lagi, maka jumlah segitiga biru akan selalu kurang dr setengahnya segitiga merah

Source: Developed by the authors

Student MA Answer (Translated)

"Yes! I agree, because the blue triangles are always located between the red triangles. So, 1 blue triangle is between 2 red triangles, 2 blue triangles are between 3 red triangles, and so on.

So, if Alex adds one more row, then the number of blue triangles will increase by 4 and the red triangles will increase by 6.

Then, the total number of blue triangles is 10 and the total number of red triangles is 15.

So every time a row is added, the number of blue triangles will always be less than half of the red triangles."

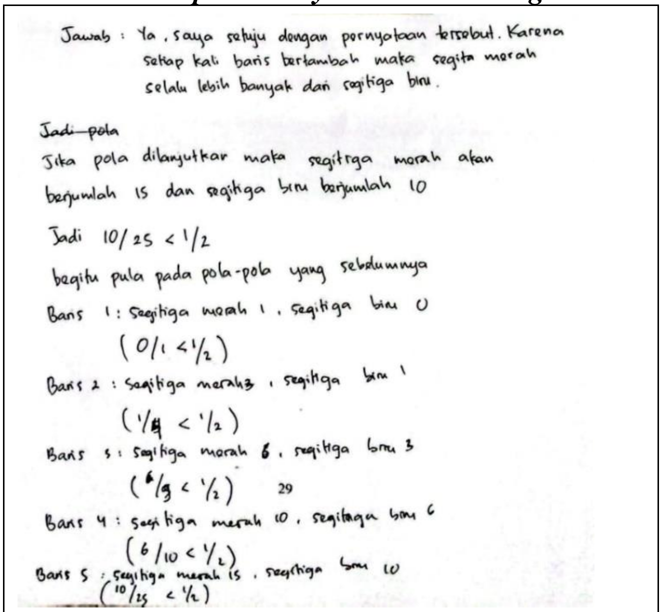
Figure 2. *Student MA Answer*

Based on the student work, MA demonstrated a solid ability to identify and explain visual patterns in the given problem. The student understood that each blue triangle was positioned between two red triangles and that the number of blue triangles was consistently fewer. Furthermore, MA accurately predicted the incremental pattern: with each additional row, four blue triangles and six red triangles were added. The student was also able to correctly compute the total number of blue and red triangles up to the fourth row, identifying 10 blue triangles and 15 red triangles, consistent with the observed pattern.

However, despite this clear understanding of the additive relationship between the triangle types, MA explanation remained largely descriptive and intuitive. The student did not yet employ formal mathematical notation to articulate that the number of blue triangles was consistently less than half of the total number of triangles. In other words, while MA successfully recognized the visual and numerical pattern, the student had not yet leveraged the concept of ratio or proportion to make a general mathematical claim.

At this stage, the teacher provided scaffolding to guide MA toward a deeper understanding of proportional reasoning. The teacher posed the question, *"What is the ratio of blue triangles to the total number of triangles in each row?"*—prompting the student to shift from a descriptive to a relational perspective. This question encouraged MA to examine the numerical relationship proportionally rather than solely visually. Subsequently, MA responded by expressing the relationship in fractional form. With this guided support, MA transitioned from intuitive observations to articulating the pattern using formal mathematical comparisons. As a result, the student was able to conclude that the number of blue triangles is always less than half the number of red triangles—a finding that reflects a successful shift from informal pattern recognition to more formal and logical mathematical justification.

4.2. RFM: *Independent Symbolic Reasoning*


Jawab : Ya, saya setuju dengan pernyataan tersebut. Karena setiap kali baris bertambah maka segita merah selalu lebih banyak dari segitiga biru.

~~Jadi pola~~

Jika pola dilanjutkan maka segitiga merah akan berjumlah 15 dan segitiga biru berjumlah 10

Jadi $10/25 < 1/2$

begitu pula pada pola-pola yang sebelumnya

Baris 1 : Segitiga merah 1, segitiga biru 0 ($0/1 < 1/2$)

Baris 2 : Segitiga merah 3, segitiga biru 1 ($1/4 < 1/2$)

Baris 3 : Segitiga merah 6, segitiga biru 3 ($6/9 < 1/2$) 29

Baris 4 : Segitiga merah 10, segitiga biru 6 ($6/10 < 1/2$)

Baris 5 : Segitiga merah 15, segitiga biru 10 ($10/25 < 1/2$)

Source: Developed by the authors

Student RFM Answer (Translated)

Answer: Yes, I agree with that statement. Because with each added row, the number of red triangles is always more than the number of blue triangles.

So – the pattern: If the pattern continues, then the red triangles will total 15 and the blue triangles will total 10. So 10/25 < 1/2.

And so on with the following patterns:

- Row 1: red triangle 1, blue triangle 0
  (0/1 < ½)
- Row 2: red triangles 3, blue triangle 1
  (1/4 < ½)
- Row 3: red triangles 6, blue triangles 3
  (3/9 < ½)
- Row 4: red triangles 10, blue triangles 6
  (6/16 < ½)
- Row 5: red triangles 15, blue triangles 10
  (10/25 < ½)

Figure 3. Student RFM Answer

Based on the student work, RFM demonstrated a strong and systematic understanding of the concept of ratio. The student was able to identify the pattern of the number of red and blue triangles in each row and subsequently calculate and compare them proportionally. Furthermore, RFM expressed the ratio of blue triangles to the total number of triangles in fractional form for each row—such as 0/1, 1/4, 3/9, up to 10/25—and consistently verified that each ratio was less than ½.

The repeated use of the comparison symbol (< ½) indicated that the student was able to generalize the pattern and understand that the number of blue triangles would always be less than half of the total number of triangles. In doing so, RFM demonstrated formal mathematical thinking and well-developed numerical skills, both in performing calculations and in drawing logical and structured conclusions.

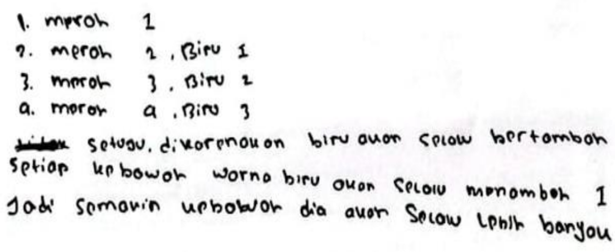
At this stage of learning, the teacher provided only minimal guidance to reinforce RFM conceptual understanding. This is illustrated in the following excerpt from their interaction:

Teacher: *"You've already written the ratio of blue triangles to the total triangles in each row. Now, how can you represent this ratio symbolically?"*

RFM: *"I can write it in fractional form and show that the fraction is less than one-half, for example, 10/25 is less than ½, and so are the ratios in the previous rows."*

This response demonstrated that RFM was capable of symbolic reasoning and independently making generalizations. Consequently, the teacher decided to fully withdraw scaffolding, as RFM showed confidence and the ability to continue learning autonomously.

4.3. IAM: *Progress through Intensive Scaffolding*



Source: Developed by the authors

Student IAM Answer (Translated)

1. red 1
2. red 2, blue 1
3. red 3, blue 2
4. red 4, blue 3

Yes, that correct, because the number of blue (triangles) will always increase.

Each row down, the number of blue (triangles) will always increase by 1. So the further down it goes, the more blue (triangles) there are.

Figure 4. Student IAM Answer

Based on the student work, IAM demonstrated an initial ability to observe changes in the number of red and blue triangles sequentially. The student was able to recognize that each additional row gradually increased the number of red and blue triangles and noted that one blue triangle was added with each new row. Moreover, IAM recognized a consistent pattern and attempted to articulate it by stating, "Each time it goes down, the blue color always increases," indicating awareness of the increasing number of blue triangles as rows progressed.

However, the student understanding remained partial, as IAM focused solely on the absolute increase in the number of triangles and had not yet developed proportional reasoning regarding the red triangles or the overall total. Therefore, the teacher provided more intensive scaffolding to guide IAM toward the concept of proportional comparison. This instructional interaction is illustrated in the following dialogue:

Teacher: *"Try to observe the ratio of blue triangles to the total number of triangles in each row. What can you conclude?"*

IAM: *"In the first row, it's 0 out of 1; in the second row, 1 out of 4; in the third row, 3 out of 9… all are less than half."*

Teacher: *"Good. So, what pattern do you notice?"*

IAM: *"The number of blue triangles is always less, less than half of the total triangles."*

This response indicated that IAM had begun to engage in symbolic thinking and was able to make a generalization independently. As a result, the teacher decided to withdraw scaffolding completely, recognizing that IAM had gained confidence and was capable of continuing the learning process autonomously.

4.4. Research Finding

The table below summarizes the key findings from the three participants (MA, RFM, and IAM).

Participant	Initial Mathematical Ability	Role of Teacher Scaffolding	Key Development and Findings
MA	Able to identify and describe visual patterns; correctly counted the number of red and blue triangles up to the fourth row, though responses were descriptive and intuitive.	The teacher provided guiding questions to prompt proportional and symbolic thinking, such as comparing the number of blue triangles to the total number of triangles.	MA successfully transitioned from visual observation to mathematical reasoning, was able to express comparisons in fractional form, and generalized that the number of blue triangles is always less than half of the total number.
RFM	Able to systematically and accurately compute and express the ratio of blue triangles to total triangles in each row (e.g., 1/4, 3/9, 10/25).	The teacher offered minimal support, mainly reinforcing the symbolic representation of comparisons.	RFM demonstrated formal and symbolic thinking from the outset, independently generalized that the number of blue triangles is always less than half the total, and showed mature and logical mathematical understanding.
IAM	Able to observe the increasing number of triangles in each row and identify the pattern of increment in blue triangles, but initially focused only on absolute changes and did not engage in ratio-based thinking.	The teacher provided intensive scaffolding to direct attention to proportional comparisons (blue triangles vs. total triangles).	IAM successfully transitioned from counting to proportional reasoning and concluded that the number of blue triangles is always less than half the total number of triangles.

Source: Developed by the authors

Overall, the three participants demonstrated different levels of mathematical thinking in recognizing and utilizing visual and numerical patterns in the red and blue triangle arrangement.

MA was initially able to identify patterns and count red and blue triangles correctly up to the fourth row, but his explanation remained descriptive and intuitive. With the teacher's scaffolding—particularly in prompting attention to the ratio of blue triangles to the total—MA gradually progressed toward proportional and symbolic reasoning. He was then able to express comparisons as fractions and to generalize that the number of blue triangles is consistently less than half the total.

In contrast, RFM displayed a high level of formal and mature mathematical thinking from the beginning. He could compute the quantities of red and blue triangles in each row and systematically express the ratio in fractional form (e.g., 1/4, 3/9, 10/25). Furthermore, RFM explicitly used symbolic comparisons (e.g., < ½) and was able to generalize that the number of blue triangles would always be less than half of the total. Given his level of understanding, the teacher's role was limited to reinforcing symbolic consistency and logical expression.

Meanwhile, IAM initially focused on absolute numerical increases in the number of triangles per row. He recognized the incremental pattern in blue triangles but did not engage in proportional reasoning. Through guided questioning by the teacher, IAM transitioned toward ratio-based thinking and was eventually able to draw the conclusion that the number of blue triangles is always less than half the total.

In conclusion, all three students demonstrated progressive development in mathematical thinking—beginning from visual and numerical observation, moving toward proportional reasoning, and ultimately employing symbolic representation to formulate logical mathematical generalizations.

4.5. Discussion

The research revealed that the three student participants demonstrated different stages of mathematical thinking development in understanding visual patterns and proportional comparisons between the number of blue triangles and the total number of triangles. These findings align with the research aim to describe the forms of teacher scaffolding in addressing students' thinking errors and facilitating their transition toward more formal mathematical understanding.

The subject MA initially exhibited the ability to recognize visual patterns and accurately count objects, but their explanations were largely intuitive and descriptive. Through interactive scaffolding in the form of guiding questions, MA was encouraged to use symbolic representations and engage with formal concepts of proportionality. This indicates that targeted teacher interventions were effective in facilitating a cognitive transition from concrete to symbolic reasoning. These findings reinforce Gidalevich &

Kramarski [2] assertion on the importance of metacognitive scaffolding in fostering conceptual understanding.

In contrast, RFM displayed a more mature level of mathematical thinking from the outset. This student was able to identify patterns, use fractions, and draw logical conclusions with minimal teacher support. As a result, the fading process could be implemented more rapidly due to RFM demonstrated learning autonomy. These findings are consistent with Brower et al. [12], who noted that students with strong initial capabilities require minimal scaffolding. In this context, the teacher's role shifted from that of an instructor to that of a facilitator who reinforces understanding.

IAM showed early recognition of object patterns but initially failed to grasp the proportional relationship. Through explicit and reflective scaffolding, IAM transitioned from simple counting to comparative analysis. This supports the notion of *contingent scaffolding* [4], whereby assistance is adaptively provided according to students' specific needs. IAM progression indicates that well-targeted scaffolding can effectively correct misconceptions and establish a solid foundation for deeper mathematical thinking.

Overall, the findings underscore that differentiated scaffolding, tailored to individual student profiles, can gradually enhance mathematical thinking. All three subjects eventually demonstrated the ability to generalize that the number of blue triangles is always less than half of the total triangles, albeit through different paths and with varying levels of teacher support. This provides empirical evidence that the effectiveness of scaffolding is highly dependent on students' initial competencies and the nature of instructional support provided.

Compared with previous literature, the current research corroborates research showing that scaffolding significantly improves conceptual understanding and learning autonomy [3], [25]. However, it also contributes novel insights through detailed documentation of the timing, types, and transitions of scaffolding in elementary mathematics instruction—a dimension that has rarely been empirically elaborated in prior research.

An unexpected yet significant finding was that IAM, who initially appeared to lag behind, made remarkable progress following precise and adaptive teacher intervention. This highlights that students' potential for mathematical thinking can flourish rapidly when supported appropriately. Consequently, the research provides important pedagogical implications: teachers should design instruction that is responsive to students' errors and plan the fading process strategically to cultivate independent mathematical reasoning.

From a theoretical perspective, this study extends the scaffolding literature by providing empirical evidence on how contingent scaffolding unfolds dynamically in response to student errors and how fading functions as a mechanism for supporting cognitive transitions from descriptive to symbolic proportional reasoning, thereby refining existing conceptualizations of scaffolding through the identification of observable indicators of support reduction that signal students' readiness for autonomy and strengthen the operational clarity of contingent scaffolding frameworks.

Practically, the findings offer actionable insights for mathematics teachers by demonstrating how scaffolding strategies can be differentiated according to students' initial thinking profiles and responsively adjusted during instruction, with the identified indicators of scaffolding reduction serving as practical guidelines for determining when instructional support should be maintained, modified, or withdrawn to foster independent mathematical reasoning in elementary classrooms.

5. Conclusion

The findings of this research indicate that contingent and adaptive scaffolding can significantly promote the development of students' mathematical thinking. All three participants demonstrated a transition from descriptive understanding to symbolic representation in identifying visual patterns, particularly in the context of proportional comparison. MA progressed from intuitive observation to symbolic reasoning through the teacher's guiding questions. RFM exhibited formal thinking from the outset and required only minimal reinforcement. Meanwhile, IAM, who initially focused on absolute numerical aspects, achieved a conceptual breakthrough following explicit instructional intervention. These findings underscore the critical role of scaffolding in addressing misconceptions and fostering mathematical generalization. By specifying observable indicators of scaffolding reduction—including decreased teacher prompting, increased student-initiated symbolic representations, and independent proportional comparisons—this study refines existing scaffolding frameworks and enhances their operational clarity for instructional decision-making.

This study recommends that teachers design flexible scaffolding trajectories aligned with students' initial cognitive profiles and systematically plan the fading process using clearly defined indicators of cognitive readiness. Such indicators can inform instructional decisions regarding when to maintain, adjust, or withdraw support. At the policy level, professional development should emphasize data-informed and student-responsive scaffolding practices. Future research should examine adaptive scaffolding in technology-enhanced learning environments and across diverse learner populations to further validate and extend these findings.

Declarations

Author Contributions

The authors confirm contribution to the paper as follows: conception and design, data collection, preparation of manuscript draft: Anton Prayitno; analysis and interpretation of results: Iis Sugiati. All authors reviewed the results and approved the final version of the manuscript.

Data Availability Statement

The data presented in this study are available on request from the corresponding author.

Funding

This research was funded by the Ministry of Education, Culture, Research, and Technology (Kemendiktisaintek) of the Republic of Indonesia.

Acknowledgements

The authors would like to thank the Heads of the Mathematics Education Department at Universitas Wisnuwardhana Malang for granting research permits and the mathematics education lecturers who contributed to the completion of this research.

Institutional Review Board Statement

This study was conducted in accordance with the Declaration of Helsinki and approved by the Institutional Review Board (or Ethics Committee) of Universitas Wisnuwardhana Malang.

Informed Consent Statement

Informed consent was obtained from all participants involved in this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this manuscript. Furthermore, all ethical considerations, including plagiarism, informed consent, misconduct, data fabrication and/or falsification, duplicate publication and/or submission, and redundancies, have been fully observed by the authors.

References

[1] OECD, Pisa 2022 Results, vol. I. 2023.

[2] S. Gidalevich and B. Kramarski, “The value of fixed versus faded self-regulatory scaffolds on fourth graders’ mathematical problem solving,” Instructional Science, vol. 47, no. 1, pp. 39–68, Feb. 2019, doi: 10.1007/S11251-018-9475-Z.

[3] J. Anghileri, “Scaffolding practices that enhance mathematics learning,” Journal of Mathematics Teacher Education, vol. 9, no. 1, pp. 33–52, Feb. 2006, doi: 10.1007/S10857-006-9005-9.

[4] J. van de Pol, M. Volman, and J. Beishuizen, “Scaffolding in Teacher–Student Interaction: A Decade of Research,” Educational Psychology Review, vol. 22, no. 3, pp. 271–296, Sep. 2010, doi: 10.1007/s10648-010-9127-6.

[5] N. Matsuda, W. Weng, and N. Wall, “The Effect of Metacognitive Scaffolding for Learning by Teaching a Teachable Agent,” International Journal of Artificial Intelligence in Education, vol. 30, no. 1, pp. 1–37, Mar. 2020, doi: 10.1007/S40593-019-00190-2.

[6] Awi, M. A. Naufal, Sutamrin, and M. Huda, “Enhancing Geometry Achievement in Pre-Service Mathematics Teachers: The Impact of a Scaffolded Flipped Classroom Using a Learning Management System,” Journal of Ecohumanism, vol. 3, no. 6, pp. 637–645, Aug. 2024, doi: 10.62754/JOE.V3I6.4035.

[7] M. F. Chen, Y. C. Chen, P. Y. Zuo, and H. T. Hou, “Design and evaluation of a remote synchronous gamified mathematics teaching activity that integrates multi-representational scaffolding and a mind tool for gamified learning,” Education and Information Technologies, vol. 28, no. 10, pp. 13207–13233, Oct. 2023, doi: 10.1007/S10639-023-11708-6.

[8] J. Zeng, P. Zhang, J. Zhou, J. Shang, and J. B. Black, “The impact of embodied scaffolding sequences on STEM conceptual learning,” Educational Technology Research and Development, vol. 73, no. 2, pp. 767–792, Apr. 2024, doi: 10.1007/S11423-024-10438-X.

[9] Y. P. Xin et al., “The Effect of Computer-Assisted Conceptual Model-Based Intervention Program on Mathematics Problem-Solving Performance of At-Risk English Learners*,” Reading & Writing Quarterly, vol. 36, no. 2, pp. 104–123, Mar. 2020, doi: 10.1080/10573569.2019.1702909.

[10] P. Ivars, C. Fernández, and S. Llinares, “A Learning Trajectory as a Scaffold for Pre-service Teachers’ Noticing of Students’ Mathematical Understanding,” International Journal of Science and Mathematics Education, vol. 18, no. 3, pp. 529–548, Mar. 2020, doi: 10.1007/S10763-019-09973-4.

[11] L. Silva, A. Mendes, A. Gomes, and G. Fortes, “Fostering regulatory processes using computational scaffolding,” International Journal of Computer-Supported Collaborative Learning, vol. 18, no. 1, pp. 67–100, Mar. 2023, doi: 10.1007/S11412-023-09388-Y.

[12] R. L. Brower et al., “Scaffolding Mathematics Remediation for Academically At-Risk Students Following Developmental Education Reform in Florida,” 2017, doi: 10.1080/10668926.2017.1279089.

[13] M. Nickl et al., “Effects of real-time adaptivity of scaffolding: Supporting pre-service mathematics teachers’ assessment skills in simulations,” Learning and Instruction, vol. 94, p. 101994, Dec. 2024, doi: 10.1016/J.LEARNINSTRUC.2024.101994.

[14] T. J. E. Faber, M. E. W. Dankbaar, W. W. van den Broek, L. J. Bruinink, M. Hogeveen, and J. J. G. van Merriënboer, “Effects of adaptive scaffolding on performance, cognitive load and engagement in game-based learning: a randomized controlled trial,” BMC Medical Education, vol. 24, no. 1, pp. 1–19, Dec. 2024, doi: 10.1186/S12909-024-05698-3.

[15] S. Rezat, S. Malik, and M. Leifeld, “Scaffolding Close Reading of Mathematical Text in Pre-service Primary Teacher Education at the Tertiary Level: Design and Evaluation,” International Journal of Science and Mathematics Education, vol. 20, no. 1, pp. 215–236, Nov. 2022, doi: 10.1007/S10763-022-10309-Y.

[16] M. P. Kshetree, B. R. Acharya, B. Khanal, R. K. Panthi, and S. Belbase, “Eighth Grade Students’ Misconceptions and Errors in Mathematics Learning in Nepal.,” European Journal of Educational Research, vol. 10, no. 3, pp. 1101–1121, 2021, doi: 10.12973/eu-jer.10.3.1101.

[17] T. Sinha and M. Kapur, “Robust effects of the efficacy of explicit failure-driven scaffolding in problem-solving prior to instruction: A replication and extension,” Learning and Instruction, vol. 75, p. 101488, Oct. 2021, doi: 10.1016/J.LEARNINSTRUC.2021.101488.

[18] J. van de Pol, N. Mercer, and M. Volman, “Scaffolding Student Understanding in Small-Group Work: Students’ Uptake of Teacher Support in Subsequent Small-Group Interaction,” Journal of the Learning Sciences, vol. 28, no. 2, pp. 206–239, Mar. 2019, doi: 10.1080/10508406.2018.1522258.

[19] I. Hidayah, T. B. Adji, and N. A. Setiawan, “Development and evaluation of adaptive metacognitive scaffolding for algorithm-learning system,” IET Software, vol. 13, no. 4, pp. 305–312, Aug. 2019, doi: 10.1049/IET-SEN.2018.5072.

[20] C. C. Chang and S. T. Yang, “Interactive

参考文:

[1] OECD, 《PISA 2022 结果》， 卷 I， 2023 年。

[2] S. Gidalevich 和 B. Kramarski, “固定与渐隐自我调节支架对四年级学生数学问题解决能力的价值,” *Instructional Science*, 第 47 卷， 第 1 期, 页 39–68, 2019 年 2 月, doi: 10.1007/S11251-018-9475-Z。

[3] J. Anghileri, “增强数学学习的支架实践,” *Journal of Mathematics Teacher Education*, 第 9 卷， 第 1 期, 页 33–52, 2006 年 2 月, doi: 10.1007/S10857-006-9005-9。

[4] J. van de Pol, M. Volman, 和 J. Beishuizen, “师生互动中的支架教学：十年的研究,” *Educational Psychology Review*, 第 22 卷， 第 3 期, 页 271–296, 2010 年 9 月, doi: 10.1007/s10648-010-9127-6。

[5] N. Matsuda, W. Weng, 和 N. Wall, “通过教授可教代理进行学习的元认知支架效应,” *International Journal of Artificial Intelligence in Education*, 第 30 卷， 第 1 期, 页 1–37, 2020 年 3 月, doi: 10.1007/S40593-019-00190-2。

[6] Awi, M. A. Naufal, Sutamrin, 和 M. Huda, “通过基于学习管理系统的支架翻转课堂提升数学师范生几何成绩,” *Journal of Ecohumanism*, 第 3 卷， 第 6 期, 页 637–645, 2024 年 8 月, doi: 10.62754/JOE.V3I6.4035。

[7] M. F. Chen, Y. C. Chen, P. Y. Zuo, 和 H. T. Hou, “设计与评估结合多重表征支架和游戏化学习工具的远程同步游戏化数学教学活动,” *Education and Information Technologies*, 第 28 卷， 第 10 期, 页 13207–13233, 2023 年 10 月, doi: 10.1007/S10639-023-11708-6。

[8] J. Zeng, P. Zhang, J. Zhou, J. Shang, 和 J. B. Black, “具身支架序列对 STEM 概念学习的影响,” *Educational Technology Research and Development*, 第 73 卷， 第 2 期, 页 767–792, 2024 年 4 月, doi: 10.1007/S11423-024-10438-X。

[9] Y. P. Xin 等, “基于计算机的概念模型干预程序对有风险的英语学习者数学问题解决表现的影响,” *Reading & Writing Quarterly*, 第 36 卷， 第 2 期, 页 104–123, 2020 年 3 月, doi: 10.1080/10573569.2019.1702909。

[10] P. Ivars, C. Fernández, 和 S. Llinares, “作为支架的学习轨迹：提升师范生对学生数学理解的观察能力,” *International Journal of Science and Mathematics Education*, 第 18 卷， 第 3 期, 页 529–548, 2020 年 3 月, doi: 10.1007/S10763-019-09973-4。

[11] L. Silva, A. Mendes, A. Gomes, 和 G. Fortes, “使用计算支架促进自我调节过程,” *International Journal of Computer-Supported Collaborative Learning*, 第 18 卷， 第 1 期, 页 67–100, 2023 年 3 月 , doi: 10.1007/S11412-023-09388-Y。

[12] R. L. Brower 等, “佛罗里达州发展教育改革后对学术有风险学生的数学补救支架,” 2017 年, doi: 10.1080/10668926.2017.1279089。

[13] M. Nickl 等, “实时适应性支架的效果：支持师范生在模拟中提升评估技能,” *Learning and Instruction*, 第 94 卷, 页 101994, 2024 年 12 月, doi: 10.1016/J.LEARNINSTRUC.2024.101994。

[14] T. J. E. Faber, M. E. W. Dankbaar, W. W. van den Broek, L. J. Bruinink, M. Hogeveen, 和 J. J. G. van Merriënboer, “自适应支架对游戏化学习中的表现、认知负荷和参与度的影响：随机对照试验,” *BMC Medical Education*, 第 24 卷, 第 1 期, 页 1–19, 2024 年 12 月, doi: 10.1186/S12909-024-05698-3。

[15] S. Rezat, S. Malik, 和 M. Leifeld, “高等师范教育中小学师范生数学文本精读的支架设计与评估,” *International Journal of Science and Mathematics Education*, 第 20 卷, 第 1 期, 页 215–236, 2022 年 11 月, doi: 10.1007/S10763-022-10309-Y。

[16] M. P. Kshetree, B. R. Acharya, B. Khanal, R. K. Panthi, 和 S. Belbase, “尼泊尔八年级学生数学学习中的误解与错误,” *European Journal of Educational Research*, 第 10 卷, 第 3 期, 页 1101–1121, 2021 年, doi: 10.12973/eu-jer.10.3.1101。

[17] T. Sinha 和 M. Kapur, “在教学前基于失败驱动支架问题解决的明确有效性：重复与扩展研究,” *Learning and Instruction*, 第 75 卷, 页 101488, 2021 年 10 月, doi: 10.1016/J.LEARNINSTRUC.2021.101488。

[18] J. van de Pol, N. Mercer, 和 M. Volman, “小组学习中学生理解的支架：学生在后续小组互动中对教师支持的吸收情况,” *Journal of the Learning Sciences*, 第 28 卷, 第 2 期, 页 206–239, 2019 年 3 月, doi: 10.1080/10508406.2018.1522258。

[19] I. Hidayah, T. B. Adji, 和 N. A. Setiawan, “自适应元认知支架在算法学习系统中的开发与评估,” *IET Software*, 第 13 卷, 第 4 期, 页 305–312, 2019 年 8 月, doi: 10.1049/IET-SEN.2018.5072。

Manuscript Information
Word count: 7,301 words (excluding references).

Peer-Review Record
Fast-track status: Not fast-tracked.
First-round reviews received: 3 reports.
Revision cycles completed: 3 rounds.
Final version submitted: January 6, 2026