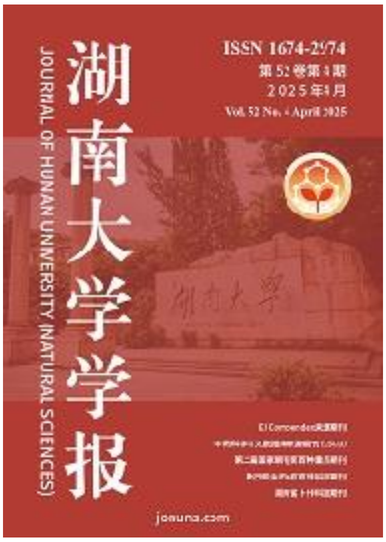

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Analytical Solution of 3D Consolidation with Mixed Drainage Boundaries using the Fourier Transform Technique

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Abstract: This study investigates the application of Fourier transform methods to address the partial differential equations (PDEs) encountered in three-dimensional consolidation issues in soil mechanics. The aim of this study is to derive a closed-form analytical solution for three-dimensional consolidation under diverse drainage boundaries and evaluate them using extensive MATLAB benchmarks. The authors illustrated how precisely selected sine and cosine expansions can convert the governing partial differential equation into a more manageable form, facilitating explicit, time-dependent solutions under three specific boundary condition scenarios: (i) all six faces drained, (ii) four side faces drained with the top and bottom faces undrained, and (iii) the top and bottom faces drained with the four side faces undrained. These scenarios illustrate how the solution framework inherently simplifies from a complete three-dimensional series to two- or one-dimensional expansions based on boundary conditions. To test and demonstrate the methodology, the authors conducted numerical examples using MATLAB to examine the temporal dissipation of the pore pressure in each situation. The results closely aligned with the theoretical assumptions and received robust approval, affirming the efficacy and adaptability of Fourier-based approaches for consolidating porous media under diverse drainage conditions. The novelty of this research is that it provides a singular Fourier-transform solution family that is effortlessly reduces to 2-D and 1-D cases, providing easily applicable formulations that avoid reliance on iterative numerical solvers.

Keywords: consolidation; Fourier transform; analytical methods; partial differential equations (PDES); pore water pressure.

基于傅里叶变换技术的混合排水边界三维固结解析解

摘要：本研究探讨了傅里叶变换法在土力学三维固结问题中偏微分方程 (PDE) 求解中的应用。本研究旨在推导不同排水边界条件下三维固结的闭式解析解，并使用丰富的 MATLAB 基准程序对其进行评估。作者阐述了如何通过精确选择的正弦和余弦展开式将控制偏微分方程转换为更易于处理的形式，从而在三种特定的边界条件情景下获得显式的时间相关解：(i) 所有六个面均排水；(ii) 四个侧面均排水，顶面和底面不排水；以及 (iii) 顶面和底面均排水，四个侧面不排水。这些情景说明了求解框架如何根据边界条件从完整的三维级数简化为二维或一维展开式。为了测试和演示该方法，作者使用 MATLAB 进行了数值计算，以检验每种情况下孔隙压力随时间的变化。研究结果与理论假设高度一致，并获得了强有力的认可，肯定了基于傅里叶变换的方法在不同排水条件下固结多孔介质的有效性和适应性。本研究的创新之处在于，它提供了一族奇异的傅里叶变换解，可以轻松地简化为二维和一维情形，从而提供了易于应用的公式，避免了对迭代数值求解器的依赖。

关键词：固结；傅里叶变换；解析法；偏微分方程（PDES）；孔隙水压力

1. Introduction

Partial differential equations (PDEs) are crucial for modeling complex systems across various domains, including engineering, physics, and finance. The solutions of partial differential equations offer substantial insights into the behavior of complex systems and their responses. Numerous numerical methods have been developed for solving partial differential equations (PDEs). Numerous investigations have recently been conducted to examine these numerical solutions. The Nonlinear Homotopy Perturbation Method (NHPM) was examined to derive solutions to systems of nonlinear partial differential equations [1]. The variational iteration technique (VIM) has been used to solve nonlinear parabolic-hyperbolic partial differential equations [2]. (HPM) was implemented with the LA transform to solve 4^{th} order nonlinear-PDEs [3]. There is a noticeable gap in the provision and discussion of analytical methods for solving PDEs [4]. Numerous analytical techniques exist for solving partial differential equations, including the separation of variables, Laplace transforms, and Fourier transforms, each possessing distinct advantages and disadvantages. For example, separation methods can address limited systems in which the separation of unknowns is applicable. The Laplace transform deals only with problems that have “t” derivative only and the initial conditions are known. One of the most powerful integral techniques for solving PDEs is the Fourier transform. It can handle higher dimensions and boundary conditions and has a wide Range [5-6]. Partial differential equations are essential for describing many problems in geotechnical engineering, particularly the consolidation process. Consolidation is a process that reduces the soil volume by releasing excess pore pressure and is a common issue in geotechnical engineering. The primary objective is to examine the settlement of foundations and determine the optimal design for various load types [7]. Since the mid-1920s, Terzaghi’s consolidation equation for saturated soils has provided a solid basis for numerous ongoing studies. Major changes in soil conditions have received considerable attention during engineering developments, profoundly affecting the reliability of the traditional soil mechanics framework. Since the early 1960s, consolidation investigations of unsaturated soils have gained attention.

An analytical solution has been discussed for the problem of one-dimensional consolidation in unsaturated soil with a limited thickness subjected to vertical loads and lateral confinement [8]. Ho [9] developed a precise analytical solution for forecasting fluctuations in excess pore-air and pore-water pressures, as well as settlement, by examining the two-dimensional (2D) plane strain consolidation of an unsaturated soil layer under various time-dependent loads. The eigenfunction expansion method has been used to derive an analytical solution for one-dimensional and two-dimensional consolidation equations under general time-dependent loading [10-11]. Numerical analyses were conducted to examine the immediate and consolidation settlements of shallow foundations resting on clayey soil [12].

In this study, a three-dimensional consolidation problem with mixed boundary conditions was investigated using a Fourier transform. In Case I, it was fully drained at all six boundaries. In Case II, it is drained at the four side faces so that the solution is reduced to the 2D consolidation problem. In Case III, it is drained on the top and bottom only so that the solution is reduced to a one-dimensional consolidation problem. Subsequently, the solution was programmed using MATLAB, and a parametric study was conducted, which was of great acceptance with the theoretical results and those from the literature. The analyzed rectangular soil block reflects the standard raft foundation and embankment shapes; its selection guarantees the direct relevance of the findings for predicting settlement durations, optimizing prefabricated-drain configurations, and directing the positioning of the monitoring piezometers in reality.

2. Fourier Transform

Fourier transform has a wide range of applications for solving complicated problems in geotechnical engineering [13-14]. The general concept behind the Fourier approach for real-world problems is as follows: To express the given differential equation, we first transform it from x-domain to $\boldsymbol{\omega}$ domain, then perform the calculations in $\boldsymbol{\omega}$ domain. Finally, we utilize the inverse transform. Figure 1 shows this process schematically.

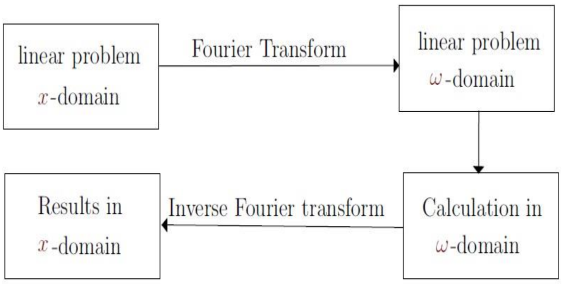

Figure 1. Technique for using the Fourier transform (Developed by the authors)

Fourier transform $F(\omega_x, \omega_y, \omega_z)$ of function $\boldsymbol{f(x,y,z)}$ can be defined as

$$F(\omega) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y,z)e^{-i(\omega_x x+\omega_y y+\omega_z z)}\,dv \quad (1)$$

Furthermore, for the inverse Fourier transform:

$$f(x,y,z) = \frac{1}{(2\pi)^3}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(\omega)e^{i(\omega_x x+\omega_y y+\omega_z z)}\,d\omega, \quad (2)$$

where

$F(\omega) = F(\omega_x, \omega_y, \omega_z)$

$dv = dxdydz$

$d\omega = d\omega_x d\omega_y d\omega_z$

When the problem in the semi-infinite domains was subjected to Dirichlet boundary conditions, the Fourier transform was reduced to a Fourier sine transform. The Fourier sine transform and its inverse can be expressed as follows:

$$F(\omega) = \int_{0}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(x,y,z)\sin\omega_x x\,\sin\omega_y y\sin\omega_z z\,dv \quad (3)$$

$$f(x,y,z) = \frac{8}{\pi^3}\int_{0}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} F(\omega)\sin\omega_x x\,\sin\omega_y y\sin\omega_z z\,d\omega \quad (4)$$

3. Three-dimensional Consolidation Problem

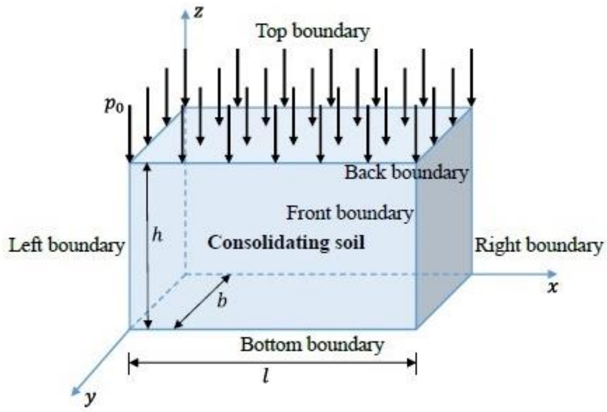

Figure 2. Three-dimensional Terzaghi's consolidation (Developed by the authors)

Figure 2 shows a three-dimensional consolidation cube with dimensions (l*b*h) subjected to an external load at its top surface and different boundary conditions, as shown in Table 1. The Fourier transform approach was used to derive the solution of the three-dimensional consolidation equation that was subjected to different boundary conditions. The governing equations for the Terzaghi consolidation problem can be defined as

$$\frac{\partial u}{\partial t} = c_v\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right), \quad u(x,y,z,0) = P_0 \quad (5)$$

Subjected to different boundary conditions as shown below in Table 1:

Table 1 Three different cases of Terzaghi's consolidation (compiled by the authors)

Case	Boundary conditions
I	$u(0,y,z,t) = u(l,y,z,t) = 0$ $u(x,0,z,t) = u(x,b,z,t) = 0$ $u(x,y,0,t) = u(x,y,h,t) = 0$
II	$u(0,y,z,t) = u(l,y,z,t) = 0$ $u(x,0,z,t) = u(x,b,z,t) = 0$ $\frac{\partial u(x,y,0,t)}{\partial z} = \frac{\partial u(x,y,h,t)}{\partial z} = 0$
III	$\frac{\partial u(0,y,z,t)}{\partial x} = \frac{\partial u(l,y,z,t)}{\partial x} = 0$ $\frac{\partial u(x,0,z,t)}{\partial y} = \frac{\partial u(x,b,z,t)}{\partial y} = 0$ $u(x,y,0,t) = u(x,y,h,t) = 0$

3.1. Case I

The solution of Case I can be obtained using the Fourier sine transform because it is subjected to drained boundaries. The distribution of the solution is as follows.

$$\frac{\partial u}{\partial t} = c_v\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) \quad (6)$$

$$u_t(\alpha,\beta,\gamma,t) = -\omega^2 c_v u(\alpha,\beta,\gamma,t), \quad (7)$$

where

$$\omega^2 = \boldsymbol{\alpha}^2 + \boldsymbol{\beta}^2 + \boldsymbol{\gamma}^2$$

$$\int \frac{u_t(\alpha,\beta,\gamma,t)}{u(\alpha,\beta,\gamma,t)} dt = -c_v \int \omega^2 dt \quad (8)$$

$$u(\alpha,\beta,\gamma,t) = u(\alpha,\beta,\gamma,0)\exp(-c_v\omega^2 t) \quad (9)$$

$$u(\alpha,\beta,\gamma,0) = \int_0^h \int_0^b \int_0^l u_0(x,y,z)\sin(\alpha x)\sin(\beta y)\sin(\gamma z)\ dv \quad (10)$$

$$= \frac{-u_0 \mathrm{lbh}}{\mathrm{mnr}\pi^3}[\cos m\pi - 1] * [\cos n\pi - 1] * [\cos r\pi - 1] = \frac{8u_0 \mathrm{lbh}}{\mathrm{mnr}\pi^3} \quad (11)$$

where

n,m,r...odd

$$\alpha = \frac{m\pi}{l}, \quad \beta = \frac{n\pi}{b}, \quad \gamma = \frac{r\pi}{h}$$

$$u(\alpha,\beta,\gamma,t) = \frac{8u_0 \mathrm{lbh}}{\mathrm{mnr}\pi^3}\exp\left(-c_v\left[\left(\frac{m\pi}{l}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{r\pi}{h}\right)^2\right]t\right) \quad (12)$$

From the inverse Fourier sine transform:

$$u(x,y,z,t) = \frac{64u_0}{\pi^3}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{r=1}^{\infty}\frac{1}{\mathrm{mnr}}\begin{bmatrix}(\sin\alpha x)\\ *(\sin\beta y)(\sin\gamma z)\\ *\exp(-c_v\omega^2 t)\end{bmatrix}, \quad (13)$$

where

$$\omega^2 = [(\alpha)^2 + (\beta)^2 + (\gamma)^2]$$

3.2. Case II

$$u(\alpha,\beta,\gamma,t) = u(\alpha,\beta,\gamma,0)\exp(-c_v\omega^2 t) \quad (14)$$

Applying Fourier transform mixed form

$$u(\alpha,\beta,\gamma,0) = \int_0^b \int_0^l \begin{bmatrix} u_0(x,y,z) \\ *\sin(\alpha x)\sin(\beta y)\sin(\gamma z)\end{bmatrix} dxdy \quad (15)$$

$$= \frac{u_0 \mathrm{lbh}}{\mathrm{mn}\pi^2}[\cos m\pi - 1][\cos n\pi - 1] = \frac{4u_0 \mathrm{lbh}}{\mathrm{mn}\pi^2} \quad (16)$$

$$u(\alpha,\beta,\gamma,t) = \frac{4u_0 \mathrm{lbh}}{\mathrm{mn}\pi^2}\exp(-c_v(\alpha^2+\beta^2)t) \quad (17)$$

$$u(x,y,z,t) = \frac{4}{\mathrm{lb}}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} u(\alpha,\beta,\gamma,t)(\sin\alpha x)(\sin\beta y) \quad (18)$$

$$u(x,y,z,t) = \frac{16u_0}{\pi^2}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{\mathrm{mn}}\begin{bmatrix}\sin\alpha x \sin\beta y \\ *\exp(-c_v(\alpha^2+\beta^2)t)\end{bmatrix} \quad (19)$$

3.3. Case III

$$u(\alpha,\beta,\gamma,0) = \int_0^h u_0(x,y,z)\sin(\gamma z)\, dz = \frac{2hu_0}{r\pi} \quad (20)$$

$$u(\alpha,\beta,\gamma,t) = \frac{2hu_0}{r\pi}\exp(-c_v\gamma^2 t) \quad (21)$$

$$u(x,y,z,t) = \frac{4}{\pi}\sum_{r=1}^{\infty}\frac{u_0}{r}\sin\gamma z \exp(-c_v\ \gamma^2 t) \quad (22)$$

The degree of consolidation at depth z can be defined as

$$U_z = \frac{u_0 - u}{u_0} = 1 - \frac{4}{\pi}\sum_{r=1}^{\infty}\frac{u_0}{r}\sin\gamma z \exp(-c_v\ \gamma^2 t), \quad (23)$$

which is the ratio of the dissipated excess pore pressure $(u_0 - u)$ to the initial pore pressure at the same point. Therefore, at $t = 0$; $U_z = 0$ and there is no consolidation. However, when $t \to \infty$, $u \to 0$, and $U_z \to 1$ (or 100%), consolidation is complete at 100%.

4. Numerical Study

A parametric study was conducted to evaluate the variation in pore water pressure in all directions for all three cases. The linear elastic behavior of the soil was assumed, and a surcharge of 100 KPa was applied to the top surface to study the variation in the pore water pressure with time. The coefficient of consolidation (Cv) was assumed to be ($1*10^{-3}\mathrm{m}^2$/day). The soil block size dimensions (10m*10m* 5). Because consolidation takes a long time to occur, analysis was performed for a period of 10^4 days so that full dissipation of pore water pressure can take place. The equations obtained for the three case studies were programmed using Matlab and Figures below show the variation in the pore water pressure over time for each case study.

Pore-water pressure refers to the hydraulic "back-pressure" present within the soil. It reduces the total stress; therefore, any change in u results in changes in the effective stress, and consequently, the strength and compressibility.

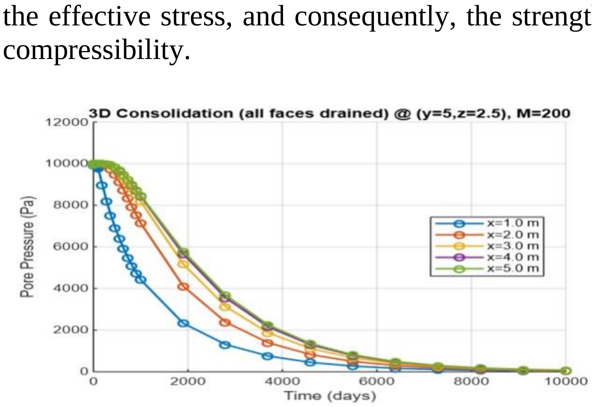

Figure 3. Variation of pore water pressure with time in the x-direction (Case I) (Developed by the authors)

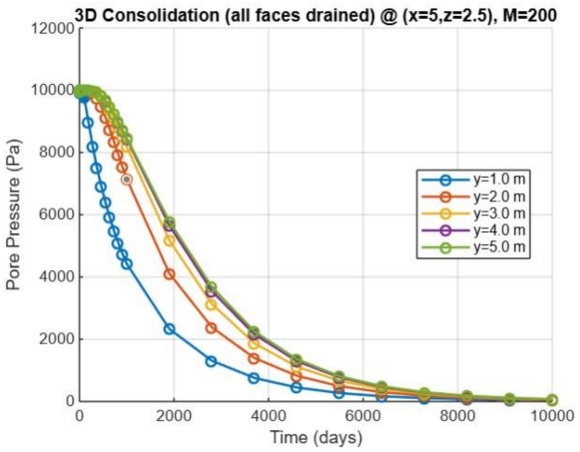

Figure 4. Variation of pore water pressure with time in the y-direction (Case I) (Developed by the authors)

Consolidation is the time-dependent transfer of the load from water to the soil skeleton as the excess pore pressure diminishes. The extent of settlement is determined by compressibility, but the rate is influenced by permeability, drainage-path length, and building sequence.

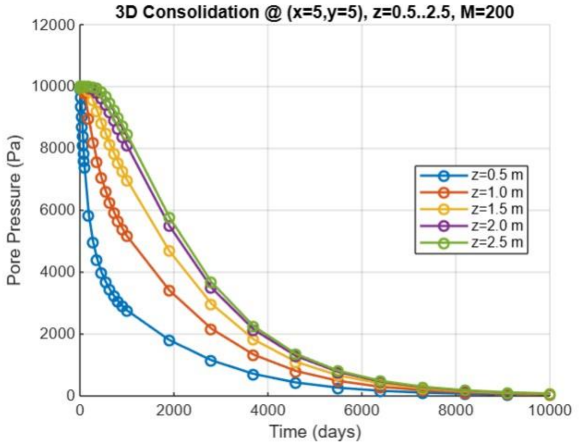

Figure 5. Variation of pore water pressure with time in the z-direction (Case I) (Developed by the authors)

The solution of Case I is programmed using MATLAB and the partial sum parameter M = 200, which means a huge triple sum (200^3=8*10^6) that is near perfect infinite sum calculations. Figure 3 shows the variation in pore water pressure with time in the x-direction. It begins at a value of P=10^4 Pa times t=0 and decreases over time. An increase in the effective stress corresponds to this decay in the pore water pressure. Thus, the sum of the pressures at any point equals the total external pressure. Because of symmetry, calculations were made to half the geometry in all directions, and the other half yielded the same results in reverse order. Figures 4 and 5 show the changes in the pore water pressure in the *y* and *z* directions, respectively. The same attitude over time for both curves begins from its peak value and diminishes as time increases until it reaches zero, which indicates full dissipation.

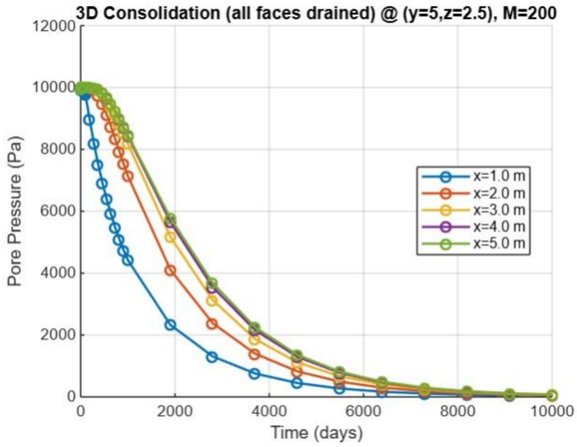

Figure 6. Variation of pore water pressure with time in the x-direction (Case II) (Developed by the authors)

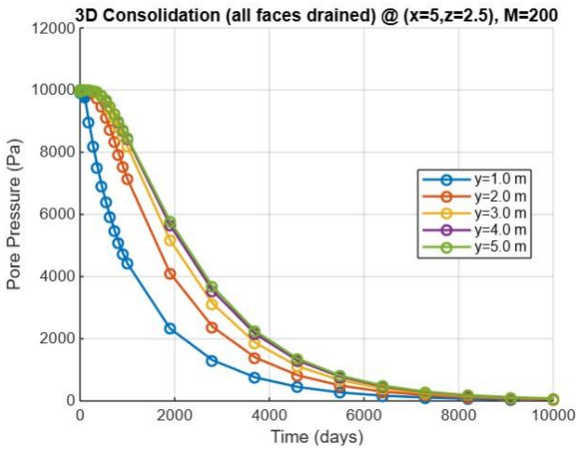

Figure 7. Variation of pore water pressure with time in the y-direction (Case II) (Developed by the authors)

In Case II, Solution did not develop any Z-variation (Figures 6 and 7), because it is undrained at (Z=0, h), the problem is reduced to a 2D consolidation problem. It is obvious that because of symmetry in the x and y directions, calculations were made in the subdomain. The center of the domain (x=5, y=5) has the highest pore water pressure over time because it is last to drain compared to the other positions.

For Case III (top- and bottom-drained and undrained sizes), the solution was reduced to a 1D consolidation problem. It depends only on the Z variation and is not affected by changes in the x- or y-direction.

Due to symmetry u(x, y, z, t)= u(x, y, 5-z, t). From the initial conditions, the pore water pressure equals zero at (Z= 0, h) and obtains its maximum value over time at the midpoint, as shown in Figure 8.

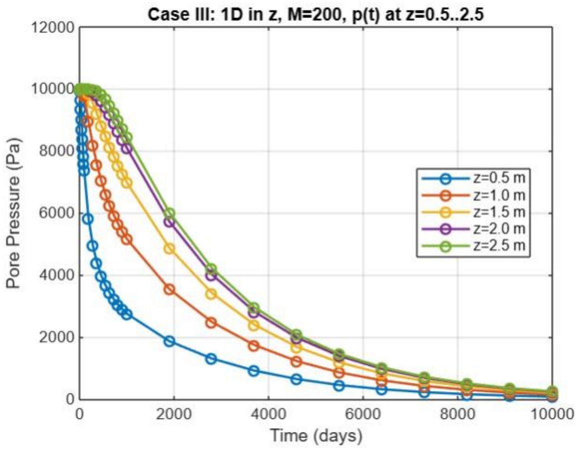

Figure 8. Variation of pore water pressure with time in the z-direction (Case III) (Developed by the authors)

5. Conclusion

This study emphasizes the significance of the Fourier transform, specifically the Fourier sine and cosine transforms as an effective and methodical approach for addressing partial differential equations (PDEs). Transforming a partial differential equation from the spatial domain to the frequency domain can simplify a complex boundary-value problem into a more manageable set of ordinary differential equations. This approach is particularly appropriate for consolidation issues in soil mechanics, where it is necessary to consider various drained or undrained (no-flow) boundary conditions. Subsequently, three potential 3D consolidation scenarios were analyzed based on a Terzaghi-type or diffusion-like formulation, each characterized by a unique set of boundary conditions:

Case I: All six faces are drained ($p = 0$ on each boundary).

Case II: Drained on all four lateral surfaces and undrained (no-flow) on the superior and inferior surfaces.

Case III: Drained at the top and bottom and undrained on the other four lateral faces.

Owing to the symmetry and consistent initial conditions, each scenario is simplified from a 3D, 2D, or 1D partial differential equation to correspondingly more straightforward Fourier expansions in one, two, or three dimensions. By precisely selecting a Fourier sine (or cosine) series that conforms to the boundary criteria, any situation can be transformed into a solvable-series solution.

These examples illustrate how Fourier-based approaches address various drainage scenarios and yield explicit time-dependent solutions for dissipating pore pressure in the soil layers. This study provides a comprehensive closed-form Fourier solution framework that includes all three mixed-drainage scenarios, thus minimizing the computational effort compared with the separate approaches found in the literature. Future work can be extended to discuss different load conditions such as time-dependent loading or more complex boundary conditions.

Declarations

Author Contributions

All authors contributed to the article structure. All the authors have read and agreed to the published version of the manuscript.

Data Availability Statement

The data presented in this study are available on request from the corresponding author.

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Not applicable.

Informed Consent Statement

Not applicable.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this manuscript.

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