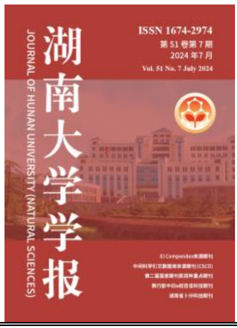

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Use of Modeling in Solving Mathematical Problems in the Moroccan Educational System

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Abstract: This article explores the dynamics of teaching and learning mathematical modeling in the first year of qualifying secondary education in the Moroccan educational context. This study focuses on the mathematical concept of "systems of equations" and seeks to identify the difficulties students encounter when modeling extra-mathematical problems. To achieve this, the analysis examines student responses to two types of problem situations: one of a classical nature and the other involving implicit unknown variables. By evaluating these responses, the objective is to identify specific obstacles students face in the modeling process. The findings revealed that rather than independently developing and applying mathematical models, students often limit themselves to adapting and manipulating pre-existing models. This indicates a gap in students' abilities to create and use models for solving real-life problems, highlighting a crucial aspect that needs to be addressed in current pedagogical methods. This study underscores the importance of improving educational approaches to enhance students' modeling skills to enhance their overall mathematical competence and ability to tackle real-life issues.

Keywords: modeling, extra-mathematical problem, system of equations.

摩洛哥教育体系中建模在解决数学问题中的应用

摘要：本文探讨了摩洛哥教育背景下合格中学第一年数学建模的教学和学习动态。本研究重点关注"方程组"的数学概念，并试图确定学生在建模非数学问题时遇到的困难。为此，分析考察了学生对两种问题情况的反应：一种是经典性质的，另一种涉及隐含的未知变量。通过评估这些反应，目标是确定学生在建模过程中面临的具体障碍。研究结果表明，学生往往局限于调整和操纵预先存在的模型，而不是独立开发和应用数学模型。这表明学生在创建和使用模型解决实际问题的能力方面存在差距，突出了当前教学方法中需要解决的一个关键方面。本研究强调了改进教育方法的重要性，以提高学生的建模技能，提高他们的整体数学能力和解决实际问题的能力。

关键词：建模、数学以外的问题、方程组。

1. Introduction

Modeling teaching and learning has been a subject of study since the early days of didactic work in France [1] and Germany [4]. Our research identified several reasons for this focus. One of the most significant reasons for this is that mathematics is viewed as a service discipline [3], and the modeling process is particularly interesting because it allows students to develop a "mathematical culture," which is crucial for training them as critical and constructive citizens, as emphasized by PISA. Furthermore, modeling is considered a vital component of mathematics teaching in international discussions. The International Conference on Teaching and Learning in Mathematical Modeling and Applications (ICTMA) provides a platform for sharing global perspectives every two years [11]. Modeling involves the interaction between mathematics and reality, encompassing aspects such as nature, culture, society, and everyday life. In the Moroccan context, the secondary education curriculum highlights the significant role that mathematical modeling plays in the teaching and learning process of mathematics [11].

Mathematical modeling poses a significant cognitive challenge for students because of the inherent complexity of tasks and specific skills required at each stage of the process. Research has highlighted the effectiveness of autonomous work in cooperative and small-group environments to facilitate the learning of modeling [12-14]. However, students face various difficulties, such as the precise comprehension of the statements, the distinction between mathematical and real models, and the interpretation and validation of mathematical results [16-18].

In this study, we are interested in exploring the teaching and learning of modeling by considering the institutional guidelines in the 1st year of secondary school in the Moroccan educational system. This system comprises a primary cycle of six levels, a secondary college cycle of three levels, and a qualifying secondary cycle of three levels. Our study falls within the framework of a very general problem relating to the "development of algebraic thinking" in Moroccan learners. More precisely, we aim to address the difficulties encountered by students in the first year of secondary school when modeling an extra-mathematical problem in a system of equations. This study focuses on the difficulties encountered by students through an analysis of their productions about problems that involve modeling using the above-mentioned mathematical concept.

Thus, we try to answer the following research questions formulated in relation to our theoretical framework:

1. Where can we find the existence of the knowledge object "modeling" in systems of equations in the mathematics program for the first year of secondary school?
2. What types of activities are considered in relation to mathematical modeling at the target level? What are the teachers' predictions about their students' responses?
3. What are the difficulties faced by students when modeling an extra-mathematical problem?

First, we present the related literature and the proposed schematization of this approach. Second, we focus on the main perspectives, "ecological analysis" and "didactic contract," to detect and understand the implications of teaching modeling. In the third part, we analyze students' productions around two situations involving modeling in systems of linear equations. Finally, we interpret the teachers' answers to a questionnaire by studying their conformity (or non-conformity) to the rules of the didactic contract already elaborated.

2. Contextualization of the Research

2.1. Conceptual Framework of the Study

2.1.1. Models and Modeling

In scientific research, particularly in the fields of experimental science, didactics, and epistemology, the concepts of models and modeling are frequently employed. It is essential to discuss various perspectives on modeling and models in scientific activity as a whole.

Modeling plays a fundamental role in the scientific activity [7] and is considered as "an interface between mathematics and biology or physics". The word "model" is used with various meanings. For example, according to one definition [9], a model may be reproduced through imitation, such as the painter's model or the model student. There are many meanings of this term in the literature. A model refers to a product resulting from modeling, whereas modeling refers to a process [5]. Models are commonly used to simplify physical realities by utilizing meaningful symbols, which are frequently encountered in everyday life to represent complex or inaccessible situations. For instance, architects model buildings for sale, stylists dress mannequins to showcase their designs, and children play with toy models of cars, trucks, and trains. These models represent idealized simplifications of reality. Mathematics education develops students' mathematical thinking skills, which are crucial for problem-solving. Mathematical models, including formulas and graphs, are employed to represent and resolve actual scenarios. They often emerge informally in classrooms, facilitating the learning process [5].

Chevallard outlines a simplified process of mathematical modeling in three steps that involve two registers of entities "a system (mathematical or non-mathematical) and a model (mathematical) of that system [1]:

Step 1: We characterize the system we aim to investigate, delineating the "aspects" that are pertinent to the examination we seek to conduct on this system, i.e., the constellation of variables through which we segment it in the realm of existence where it is discernible to us.

Step 2: The model is then constructed by establishing a certain number of relations between the variables considered in the first step, with the model of the system to be studied being the set of these relations.

Step 3: The model obtained is worked on, with the aim of generating knowledge related to the system studied, which takes the form of new relationships between the variables of the system.

Step 3 is the only phase in Chevallard's perspective that is inherently mathematical [1].

Building on Chevallard and Henry's research, Coulange [2] presents the concept of a "pseudo-concrete model" as introduced by Henry. This model is defined as "an interpretation of the system being studied in terms of natural language, while the underlying mathematical structure of the associated mathematical model is more or less implicit. Therefore, the transition from the pseudo-concrete model to the mathematical model involves translating into a mathematical language in relation to a selected mathematical framework." The approach established by Coulagne contains four interrelated phases:

Phase 1: Passage from a real-life departure to a pseudo-concrete model

Phase 2: Passage from the pseudo-concrete model to the mathematical model

Phase 3: Purely mathematical work in the mathematical model

Phase 4: Return to the studied situation

In this study, modeling an extra-mathematical problem as a system of equations is a three-step process. The first step is the equation formulation, the second step is the mathematical solving of these equations, and the final step is returning to the initial situation.

2.2. Theoretical Framework of the Study

2.2.1. Ecological Analysis

In accordance with Chevallard's [6, 8] perspective, teaching objects are portrayed as having distinct roles in relation to one another; they are considered to be living objects that can undergo changes due to the influence of other objects with which they interact. From a theoretical standpoint, he defined the ecological analysis of an object of knowledge by highlighting two key concepts: the habitat, which signifies the locations where the object exists and its associated conceptual environment, and the niche, which refers to the object's role within the system of objects it interacts with.

2.2.2. Didactic Contract

The didactic contract, as proposed by Brousseau [15], plays an important role in analyzing the behaviors of teachers and students in a modeling approach. This implicit relationship establishes the rights and responsibilities of both parties during mathematical knowledge acquisition. According to Chevallard [2], the didactic contract encompasses the constraints imposed by the institution on the treatment of knowledge, which impacts the evolution of personal reports toward an institutional report. In the Moroccan educational system, it is possible to determine the rules of the didactic contract that relate to the process of modeling.

3. Method

Our methodology, as indicated in the introduction, aims to explore the teaching and learning of mathematical modeling in the context of the first year of Moroccan secondary education, focusing on the difficulties encountered by students when modeling extra-mathematical problems using equation systems. The research begins with a review of the literature on modeling in mathematical education. Next, ecological analysis and the concept of a didactic contract are used to understand the institutional and pedagogical issues of modeling teaching, examining how modeling knowledge is integrated into the educational environment and curriculum. Data collection includes the analysis of students' productions on problems involving modeling with equation systems to identify common difficulties, as well as questionnaires distributed to teachers to gather their insights into students’ responses and understand how these responses align with the expectations of the teaching contract. The findings from the analysis of student work and the feedback provided by instructors are subsequently combined to formulate conclusions about modeling teaching and learning, with a focus on evaluating the alignment of current instructional methods with curriculum goals and the provisions outlined in the teaching contract.

3.1. Ecological Analysis of the Program and Textbooks

3.1.1. Studies on Educational Orientations and Textbooks

- *Secondary cycle*

The latest reforms in secondary school mathematics instruction, which were implemented in November 2007 [10], place a significant emphasis on solving mathematical and extra-mathematical problems. The new textbooks include activities that involve modeling practices and the construction of new knowledge objects. This new approach aligns with the problem-solving-based organization. The pedagogical orientations emphasize the significance of cultivating disciplinary competencies that enable students to engage in various mathematical modeling functions.

Consequently, in the general aims of mathematics instruction, particularly in the second and third objectives of "developing the student's capacity to solve issues and to communicate mathematically," it is highlighted that it is essential to foster the student's ability to model situations [10]: "to enhance students' problem-solving abilities; to cultivate their capacity to identify problems from both familiar and unfamiliar mathematical situations and articulate them through mathematical models."

In these educational orientations, it can be observed that there are additional modeling functions that play a crucial role in enhancing students' abilities at different levels, as evidenced by the translated and excerpted passages from the official Moroccan pedagogical guidelines for the secondary cycle [10]: "development of students' ability to model situations, present demonstrations, explain strategies, or solve problems using oral and written expressions, drawings, graphs, or algebraic methods; develop their ability to practice mathematical discovery using appropriate models; acquire essential knowledge and skills in different branches of mathematics; provide students with sufficient mathematical knowledge and skills for their future studies or integration into the work field."

In summary, our initial analysis of the program revealed that although various components are present, they are not sufficiently elucidated, and there is a dearth of descriptions that adequately explain the nature of this approach. This lack of clarity may hinder educators' ability to fully comprehend and utilize the program's potential benefits.

- *Two textbooks chosen*

Ecological analysis of teaching texts explores how the universe of taught knowledge is institutionally delimited. This involves analyzing the interrelationships between objects, sub-objects, and super-objects in the teaching text, highlighting their habitats, ecological niches, trophic levels. Our choice was to study the following two textbooks because they are most commonly used by the majority of teachers according to the results of a survey: Najah in mathematics and Almoufid in mathematics. As long as our work objective is to study modeling by a system of equations in extra-mathematical problems, we find that through textbooks, we can see the assumptions about the rules of the didactic contract that are related to modeling.

The habitat for modeling through systems of equations in the Najah and Mofid textbooks is the course titled "First-Degree Equations and Inequalities - Systems," for which we provide more details in the tables below.

Table 1 Analysis results for the two textbooks (The authors)

Aspect	Najah Textbook	Almoufid Textbook
Expected Abilities	1. Solving systems of the first degree with two unknowns using different methods (linear combination, substitution, and determinant) 2. Mathematicizing situations with variable quantities using systems	Not mentioned
Preparatory Activity	**Activity 6** - Solving a system given directly - Solving a problem leading to solving the given system in the first question - More complicated activity with no resolution guidelines	**Activity 4** - Identifying and representing two unknowns using different variables - Translating the situation by a system of equations - Determining the solution
Course Content	- No explanation on problem-solving - No examples of problem types	- No explanation on problem-solving - No examples of problem types - Method of the determinant presented with an example. Other methods are prerequisites.
Solved Exercises	- No examples of problem-solving or problem types	Example 4: Mathematization of a situation with systems - Simple example: unknowns explicitly given in the question - Validation is not performed.

Table 2 Exercises and problem analysis in the two textbooks (The authors)

Textbook	Total Exercises	Extra-Mathematical Exercises	Comments
Najah	21	4	Mathematical work on pre-constructed models
Almoufid	14	2	Selecting correct quantities for modeling; validation steps are often absent.

Table 3 Examples of extra-mathematical exercises in the two textbooks (The authors)

Textbook	Exercise Title	Statement	Comments
Najah	Weight of Three Friends	"Salwa and Ahmed together weigh 60 kg, Salwa and Omar weigh 39 kg, and Ahmed and Omar weigh 45 kg. What are the weights of Salwa, Ahmed, and Omar?" (p. 106)	Explicit unknowns stated in the question

Textbook	Exercise Title	Statement	Comments
Almoufid	Price of Books	"The price of 12 books in mathematics and 7 books in physics is 264 Dh; for 8 books in mathematics and 14 books in physics, the total price is 288 Dh. Calculate the price for each book."	Quantities to be taken as unknowns explicitly announced

Based on our analysis of the two textbooks, mathematical modeling through systems of equations is presented as a method for addressing problems in the habitat that involve systems of equations. This suggests that there may not be a comprehensive approach to modeling both intra- and extra-mathematical problems as the focus seems to be on solving problems using pre-existing models and resolution tools.

The methodology outlined in the Almoufid manual's discovery activity is comprised of three steps. Here, we propose an additional step, inspired by Coulagne's modeling approach, which involves returning to the original problem at hand. Consequently, we suggest a four-step modeling approach:

1. Distinguish the two unknowns and assign them separate variables.
2. Translate the situation into a system of equations.
3. Solve for the unknowns.
4. Return to the original problem.

In the remainder of this paper, we refer to this approach as "the θ approach."

Based on the analysis of both manuals, it can be concluded that no identifiable niches were discovered in other disciplines. After completing all the exercises proposed in both manuals, particularly those related to systems of equations in the course titled "Equations and Inequalities of the First-Degree Systems," it is evident that most of the exercises possess the following characteristics:

1. Most problems and exercises cover a system of two equations with two unknowns.
2. Most exercises have quantities explicitly presented in the statement.
3. The unknowns conform to these quantities.
4. In most of the exercises, unknowns are explicitly stated in the questions.

The analysis demonstrated that the teaching of modeling fails to achieve the curricular objectives outlined in the "Official Educational Orientations" document, specifically "development of the student's ability to practice mathematical discovery based on suitable models" and "the student's ability to model situations" and that the modeling of problems, whether of an intra- or extra-mathematical nature, is confined to a method of equating that involves selecting appropriate information to develop a mathematical model that represents the situation. This model is merely a condensed representation of non-mathematical entities (represented in language) in mathematical notation.

3.2. Rules of the Didactic Contract

Based on the findings of the feasibility analysis, we can formulate hypotheses regarding the didactic contract rules that can be used to explain the behavior of students in modeling non-mathematical problems in this class. These rules are derived from the exercises assigned to the "Equations and Inequations of the First Degree System" chapter, which pertains to systems of equations.

Rule 1: To solve an extra-mathematical problem, it is necessary to considering writing a system of equations.

Rule 2: The quantities to be chosen as unknowns are explicitly stated in the statement.

Rule 3: The quantities to be chosen as unknowns are explicitly listed in the question at the end of the statement.

Rule 4: A relationship between two variables must be established to write an equation.

Rule 5: The system to be written from the statement must be solvable by the method of the determinant observed in the course or by one of the other methods considered prerequisites for the student.

The didactic contract's tenets serve as a means to address inquiries related to our problematic. To evaluate their effectiveness, we devised an assessment for students and a questionnaire for educators.

3.3. Experimentation

3.3.1. Questionnaire for Teachers

The development of a tailor-made questionnaire was undertaken specifically for the 53 online teachers in their first year of secondary school education. Its aim was to evaluate whether their conduct conforms to the guidelines set forth in the didactic contract. The questionnaire comprised two sections: the initial section focused on queries related to textbooks most commonly used by the teachers; the second segment presented two statements of exercises that were part of the experimentation, with the intention of gauging the teachers' opinions on which exercises they believed their students in their respective classes would be capable of solving and on which ones they would not, along with the reasons for their beliefs. For every scenario, we inquired whether their students would be able to resolve the situation in their class (i.e., whether an expected resolution was possible), and we also requested their opinions on the matter using the "other" box. Prior to asking these questions, we inquired as to whether these types of situations were addressed in their classes or not.

3.3.2. Experimentation

The work of Henry falls within the same category as he presented the concept of the "pseudo-concrete model," which serves as an intermediary between reality and mathematical models. This model represents the first level of abstraction for what is referred to as a "real situation." To fulfill the objectives of this study and to determine whether the students' behavior adheres to the rules of the didactic contract, we devised an experiment for first-year secondary school students. This experiment was conducted in four classes across two high schools located in two cities in Morocco. The sample comprised 122 students.

This experimentation consists of two exercises, each in the French and Arabic languages, and has a time frame of 40 minutes. The first exercise is a well-known problem that adheres to all the didactic contract rules established and can be solved using the stages of the θ approach introduced above. This exercise is sourced from the Alnajah manual. The second problem is inspired by an exercise posed in a third-year class during a didactic study of mathematics [15]; it requires the exploration of implicit unknowns. Further details can be found in Table 4.

Table 4 Presentation of the two situations given in the experiment (The authors)

Aspect	**Florist 1**	**Florist 2**
Statement	A woman bought two bouquets of flowers: The first bouquet contains one rose and five iris flowers for 17 dirhams. The second bouquet contains three roses and two iris flowers for 25 dirhams. What is the price of a rose? What is the price of an iris flower?	A florist sells rose bouquets. From Monday to Saturday, she offers her bouquets of roses at a fixed price and applies a separate reduced price for the Sunday market, which remains consistent from week to week. During the initial week, she offers 65 bouquets at the standard price and 18 bouquets at the lower price on Sunday. Her weekly profit is then 939 dirhams. During the second week, she offers 52 bouquets at the regular price and 12 bouquets at the reduced price on Sunday. As a result, her weekly profit amounts to MAD 756. Is the reduced price a gain or loss for the florist? How much?
Problem Type	Classical problem representing a daily life situation	Non-classical problem; the difficulty lies in the choice of unknowns.
Steps of the θ Approach	1. Choose unknowns indicated in the question (x = price of a rose, y = price of an iris flower) 2. Link unknowns to write equations $\begin{cases} x + 5y = 17 \\ 3x + 2y = 25 \end{cases}$ 3. Solve the system 4. Return to the problem to state prices	1. Select unknowns (x = profit at a regular price, y = profit at a reduced price) 2. Link data to write equations: $\begin{cases} 65x + 18y = 939 \\ 52x + 12y = 756 \end{cases}$ 3. Solve the system 4. Return to the problem to state if the reduced price is gain or loss (y = -2)
Difficulties	No major difficulties expected; following didactic rules	Potential difficulties: - Choice of unknowns (price, gain, and loss) - Handling negative results (reduced price as a loss) - Revisiting calculations due to unexpected results (negative value for y)

The test was designed for students to solve the problems individually. They were provided with instructions at the top of the test to ensure understanding. The students were required to complete the entire test on the provided sheet, including the draft, research steps, and final answer. Crossing out on the sheet was allowed as long as all research steps were left visible. If a student was unable to solve a problem, they were instructed to indicate the difficulties in Arabic or French that prevented them from completing the test.

Through the analysis of students’ productions, we focus on modeling systems of equations and not on the methods of solving them or solving the systems themselves.

4. Results and Discussion

4.1. Analysis of Questionnaire Responses

We received responses from 53 teachers who teach the first year of secondary education.

Most teachers use textbooks to prepare lessons (92.5%) or propose exercises to their students (84.9%).

For the first question "Do you use situations of modeling by equations or system of equations?," we gave the first situation of our experimentation as an example; the answers were as follows: 38 teachers answered “yes”; the others answered “no”; the second question is the same as the first but the example this time is the second situation of our experimentation; 54.7% declared that they do not use such situations in their classes.

The answers to the question "Do you think that your students in your classes will be able to solve them?" were diverse because only 11 (20.8%) teachers answered “yes,” 33 (62.3%) teachers answered “no,” and the others used the box "other" to add their personal opinions on it; we can address their remarks, which show difficulties, especially in the second exercise:

Achref: Unfortunately, the majority are not able to do it.

Monim: Some students.

Zobair: They will be able to solve the first exercise. For the second one, they will find difficulty in understanding, but some elements (1 or 2) can help them answer.

Ahmed: Some students can find the solution, but most of them cannot find it.

Mohamed: Most students avoid reading texts in French because of the difficulty they find. So, for me, it is to be avoided. I propose such situations as free work for those who want to.

Anas: They can model this situation in a system; however, they will find a problem in interpretation.

Based on the responses received for the question "Of the two exercises provided, which exercise do you think most students will work on?", it was found that 86.3% of the teachers indicated that they believed the first exercise would be completed by the majority of students, while 13.7% of the teachers indicated that they believed the second exercise would be more commonly completed. The data suggest that the majority of teachers expect the first exercise, which was presented in the experiment, to be the one most students will complete, while the second exercise will be completed by fewer students. The following diagram illustrates this distribution of responses.

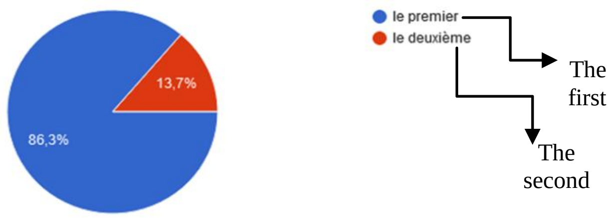

Fig. 1 Results on the question “Which problem will the students work on the most?” (The authors)

According to this survey, the expectations of the majority of teachers confirmed that the second situation represents a challenge for most students because it represents a break from the classical exercises suggested in textbooks.

4.2. Analysis of Student Productions

We typically regard an accurate response as one in which the student correctly identifies the unknowns, develops an appropriate system, resolves it, and ultimately provides the solution. In the worst-case scenario, the student constructs the system without specifying the values of x and y, develops the system, resolves it, and presents the solution, while clarifying the meaning of x and y and assigning each value to its corresponding magnitude in the statement.

We typically consider an incomplete response to be one in which the student makes the correct selection of unknowns, develops the system, but either fails to solve it or neglects to provide the solution. We generally categorize an incorrect response as one that omits any of the steps in the θ process, contains an incorrect choice of unknowns or calculation errors, or neglects any of the steps in the θ process.

4.2.1. Students’ Productions in Response to the "Florist 1" Exercise 1

Twelve students, who submitted incorrect responses, failed to indicate the variables and did not revisit the problem; instead, they directly solved the system without specifying the meanings of x and y. The remaining incorrect responses (35 students) were the result of calculation errors. Additional details can be found in Table 6.

Table 5 Results related to students’ productions in Problem 1 (The authors)

	Correct answers	**Incomplete answers**	**Incorrect answers**	**No answers**
Number	74	1	47	0

Table 6 Results related to students’ productions on Problem 1 according to the steps of approach θ (The authors)

Step of the θ Approach	**Number of Students Who Succeeded**	**Number of Students Who Failed**	**Comments**
Step 1	110	12	12 students did not specify unknowns x and y
Step 2	110	12	12 students did not identify x and y
Step 3	86	24	1 student did not complete the test; 23 calculation errors
Step 4	74	12	12 students did not return to the problem.

4.2.2. Students’ Productions in Response to the "Florist 2" Exercise 2

We observed that 33 students did not attempt to address this problem. We present the findings of our research on the written output of 122 students, which was conducted in accordance with the methodology set forth in our approach.

Table 7 Results related to students’ productions on Problem 2 (The authors)

	Correct answers	**Incomplete answers**	**Incorrect answers**	**No answers**
Number	1	0	88	33

Table 8 Results related to students' productions on Problem 2 according to the steps of approach θ (The authors)

Step of the θ Approach	**Description**	**Number of Students**	**Comments**
Step 1	Only one student chose the right unknowns (x as profit at a normal price and x-y as profit at a reduced price). 14 students considered x as a normal price and y as a reduced price. 33	Correct: 1 Incorrect: 14 No Answer/Confused: 33	Most students struggled to identify the correct unknowns.

	students did not answer or wrote remarks such as "It is not understood," "It is a little complicated," or "I did not understand the problem well."		
Step 2	Thirty nine students gave system (S) $\begin{cases}65x+18y=939\\52x+12y=756\end{cases}$ with x representing a normal price and y representing a reduced price. One student gave the correct system (S') $\begin{cases}64x-12y=756\\83x-18y=939\end{cases}$ 30 students gave system (S) without identifying x and y.	Correct System with Identity: 1 Correct System without Identity: 30 Incorrect System: 88	Majority were unable to correctly identify or translate unknowns.
Step 3	15 students solved system (S), whereas 22 of the 70 students who wrote system (S) (with or without the identification of x and y) solved it. Others made calculation errors or did not complete the task. One student changed the sign of the negative value of y, indicating doubt about the calculation.	Correct: 15 + 22 = 37 Errors/Unfinished: 33	A significant number of students had calculation errors or left the problem unfinished.
Step 4	Nineteen students answered the final question. One student who correctly answered identified that the reduced price was a loss of 2 dirhams (y = -2). 10 students indicated the price was a loss, 3 did not return to the problem despite finding x and y, and others mistakenly thought the reduced price was a gain.	Correct: 1 Loss Identified: 10 No return: 3 Incorrect gain: 5	Mixed responses, with many students misunderstanding the final implications

These results confirm the teachers' predictions regarding this issue and highlight how this problem diverges from the norms of the didactic contract associated with solving extra-mathematical problems through equations. The students' practices demonstrate their capability to solve a system of two equations using methods deemed prerequisites or the determinant method taught in class. However, they demonstrate limited ability to transcend the conventional framework of classic problems where the variables to be considered unknowns are explicitly defined in the question at the conclusion of the problem statement.

5. Conclusion

This study explored the teaching and learning of mathematical modeling, focusing on linear systems as a tool for equation-based modeling in the first year of qualifying secondary education. Based on our research questions related to mathematical modeling at this level, we conducted an ecological analysis of the Moroccan educational system and examined the didactic contract associated with the approach to equation formulation.

Our findings reveal that widely used textbooks do not devote sufficient attention or interest to modeling extra-mathematical problems. It is evident that the current approach does not constitute true modeling but rather a method of developing equations to solve problems considered "extra-mathematical." We refer to this method as the θ approach. Although modeling holds a significant place in the curriculum, most activities and situations proposed in textbooks do not require the construction of mathematical models. Mathematizing most problems involves a straightforward translation of information from the verbal domain to the mathematical domain without examining the validity of the model.

In our evaluations of students' work, we found that they consistently adhere to the guidelines of the didactic contract with regard to the translation of extramathematical problems into equations. Their task is confined to constructing the appropriate system of equations based on the given statement. The students view modeling as a practical exercise that employs a well-known and unchanging model, which follows predetermined didactic contract rules. The expectations of teachers align with the behaviors of students in relation to the two test problems. This prompts us to contemplate the past pedagogical experience of both teachers and students, particularly in the third year of middle school, where the system of equations was first introduced. It would be greatly beneficial to investigate the instruction of modeling with linear systems in the third year of middle school and the pedagogical practices associated with modeling.

We recommend revising teacher training programs to include more authentic and effective modeling strategies. It is essential to develop educational tools that encourage students to create and validate their own mathematical models rather than merely manipulating pre-existing models. In terms of research perspectives, extending this study to other educational levels, particularly the third year of middle school, where systems of equations are initially introduced, would be valuable. Investigating teachers' pedagogical practices at this level could provide valuable insights for improving mathematical modeling teaching in the early years of secondary education.

In summary, this research offers a new perspective on mathematical modeling teaching, contributing to the enhancement of pedagogical practices and the optimization of educational outcomes within the Moroccan context. Our findings underscore the need for ongoing teacher training and revision of textbooks to include more effective and authentic modeling approaches.

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