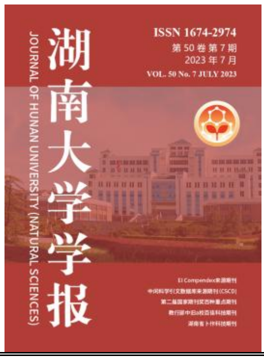

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Analysis of Convergence and Noise Terms in the Adomian Decomposition Method for Nonlinear PDEs

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Abstract: In this study, we enter the field of nonlinear partial differential equations (PDEs) and propose a novel approach to their solution. We employ the iterative Adomian decomposition method (ADM) to address a distinct subclass of nonlinear PDEs. The main objective of this paper is to evaluate the efficiency and reliability of the ADM in handling complex mathematical models. To this end, we explore the convergence characteristics of the method when applied to the aforementioned class of equations. Our analysis sheds light on the intricate interplay between the iterative steps of the method and the underlying nonlinear dynamics, providing valuable insights into the behavior of the solution process. Through this investigation, we find noise terms in the context of inhomogeneous equations. We reveal that the presence of noise is intrinsically linked to the inherent heterogeneity of the equations, and its impact on convergence behavior is systematically elucidated. To validate the robustness and accuracy of the proposed method, we present a series of numerical results. These results not only underscore the reliability of the method in producing accurate solutions but also highlight its potential to overcome computational challenges associated with nonlinear PDEs. In summary, our study contributes to the broader picture of nonlinear PDE solvers by showing the potential of the Adomian decomposition method, especially in models involving noise terms and inhomogeneous equations. The efficiency of this method is illustrated by investigating the convergence results for this equation. We show that the noise terms are conditional for nonhomogeneous equations, and the numerical results demonstrate the reliability and accuracy of the ADM.

Keywords: the Adomian method, noise terms, iterative method.

非线性偏微分方程亚多米安分解法的收敛性和噪声项分析

摘要：在这项研究中，我们进入非线性偏微分方程（偏微分方程）领域，并提出了一种新的解决方法。我们采用迭代阿多米亚分解方法(ADM)来解决非线性偏微分方程的独特子类。本文的主要目的是评估ADM在处理复杂数学模型时的效率和可靠性。为此，我们探讨了该方法应用于上述一类方程时的收敛特性。我们的分析揭示了该方法的迭代步骤与潜在的非线性动力学之间复杂的相互作用，为求解过程的行为提供了有价值的见解。通过这项研究，我们发现了非齐次方程中的噪声项。我们揭示了噪声的存在与方程的固有异质性有着内在的联系，并且系统地阐明了它对收敛行为的影响。为了验证所提出方法的鲁棒性和准确性，我们提出了一系列数值结果。这些结果不仅强调了该方法在产生准确解决方案方面的可靠性，而

且还强调了其克服与非线性偏微分方程相关的计算挑战的潜力。总之，我们的研究通过展示亚多米安分解方法的潜力，特别是在涉及噪声项和非齐次方程的模型中，为更广泛的非线性偏微分方程求解器做出了贡献。通过研究该方程的收敛结果说明了该方法的效率。我们证明噪声项对于非齐次方程是有条件的，数值结果证明了ADM的可靠性和准确性。

关键词：亚多米安方法、噪声项、迭代方法。

Introduction

The Adomian decomposition method (ADM) is a well-known semi-numerical technique known for its effectiveness in solving ordinary and partial differential equations (ODEs and PDEs) [1]. Originally conceptualized by Adomian, this method introduces a distinctive approach by developing an analytical solution in the form of a polynomial series. Unlike traditional methods, ADM offers an alternative way to obtain analytic series solutions for differential equations, often reflecting the Taylor expansion of the true solution around the reference point $x_0 = 0$. Importantly, the method demonstrates rapid convergence within a localized region, underscoring its potential for accuracy in solving complex equations.

One of the most important phenomena of the Adomian method is the notion of noise terms. These noise terms bring substantial advantages by facilitating accelerated convergence of the solution, a feature that is especially pronounced in homogeneous PDEs. These noise terms manifest themselves as distinct but interconnected expressions that materialize in various components of the series. In particular, an interesting pattern emerges when terms within iterations cancel each other out, and subsequent iterations comprise those terms that persisted beyond the initial cancelation. Consequently, the accumulation of these non-annulling terms ends up in the solution of the equation itself [2].

This relationship between noise terms and iterative cancelation gives the Adomian decomposition method an interesting ability to efficiently tackle more complex nonlinear differential equations. Importantly, the phenomenon of noise terms not only accelerates the convergence process but also offers a view through which to visualize the underlying behavior of the solution of the equation [3, 4]. This deep insight into the ADM goes beyond a computational technique, positioning it as a powerful tool for unraveling the complexities of nonlinear ODEs and PDEs. Through empirical validation and theoretical analysis, this study demonstrates the scope of the Adomian Decomposition Method as a versatile approach with applications spanning diverse fields, including physics, engineering, and mathematical modeling.

1. Description of the ADM

In the field of solving linear and nonlinear differential equations, the ADM represents a very important breakthrough because it offers a mathematical framework different from known methods in the literature for dealing with ordinary and partial differential equations (ODEs and PDEs) [6-8]. The method was developed by George Adomian, which introduces a novel approach combining semi-analytical and numerical techniques, showing his proficiency in dealing with complex mathematical problems.

In essence, Adomian's method is based on the fundamental principle of decomposing a complex differential equation into a series of simpler sub-equations [9-11]. The decomposition is achieved by iteratively separating the equation into additive components, each of which can be solved independently. The iterative process proceeds so that the solution of the original equation emerges as an accumulation of solutions of these new equations. The main feature of the Adomian method is the presentation of the solution as a polynomial series comprising an infinite sum of terms. Each of these terms corresponds to a unique component of the solution and is constructed using Adomian polynomials. The polynomials, computed recursively from the original equation and the new equations, frame the analytical essence of the solution, capturing the intricate interplay of the system dynamics [12-14].

In the application of this method, the original differential equation is expressed in the form of an operator acting on an unknown function. The equation is then systematically decomposed, resulting in a recursive sequence of equations. Each of these equations encapsulates progressively higher-order derivatives and iteratively encapsulates the components of the solution, gradually building up the complete serial solution.

Although the Adomian method does not necessarily provide a closed-form solution, its serial representation provides insight into the behavior and properties of the solution [15-18]. Convergence analysis is crucial to ensure the validity and accuracy of the obtained solution. The convergence properties of the ADM are influenced by several factors, such as the equation complexity and decomposition choice [19, 20].

In the following sections, we delve into the methodology of the method, presenting the iterative procedure step by step, discussing its convergence characteristics, and highlighting its versatility in handling nonlinear differential equations. Through mathematical exploration and illustrative examples, we aim to provide a comprehensive understanding of the ADM and its importance in the field of mathematical problem solving [21, 22].

The ADM is applied to a nonlinear equation [5].

$$Lu + Ru + Nu - g = 0 \tag{1}$$

where the linear terms are decomposed into $L+R$ and the nonlinear terms are represented by Nu. Here, L is the operator of the highest-ordered derivatives with respect to t and R is the remainder of the operator. Thus, we obtain

$$Lu = -Ru - Nu + g \tag{2}$$

Here, there is an inverse operator L^{-1} of L defined by

$$L^{-1}(\cdot) = \int_0^t \int_0^t (\cdot)\ dt\, dt \tag{3}$$

Therefore, if L is a second-order operator, L^{-1} is defined by a two-fold indefinite integral

$$L^{-1}Lu = u(x,t) - u(x,0) - t\frac{\partial u(x,0)}{\partial t} \tag{4}$$

Now, operating on both sides of Eq. (1) by using L^{-1} we obtain

$$L^{-1}Lu = L^{-1}g - L^{-1}Ru - L^{-1}Nu \tag{5}$$

We have

$$u(x,t) = u(x,0) + t\frac{\partial u(x,0)}{\partial t} + L^{-1}g - L^{-1}Ru - L^{-1}Nu \tag{6}$$

The ADM represents the solution of Eq. (6) as a series

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) \tag{7}$$

The nonlinear operator Nu can be decomposed as follows:

$$Nu = \sum_{n=0}^{\infty} A_n \tag{8}$$

Therefore, substituting (7) and (8) into (6) yields

$$\sum_{n=0}^{\infty} u_n(x,t) = u_0 - L^{-1}R\sum_{n=0}^{\infty} u_n(x,t) - L^{-1}\sum_{n=0}^{\infty} A_n \tag{9}$$

where

$$u_0 = u(x,0) + t\frac{\partial u(x,0)}{\partial t} + L^{-1}g \tag{10}$$

Then, we can obtain

$$u_1 = -L^{-1}Ru_0 - L^{-1}A_0 \tag{11}$$

$$u_2 = -L^{-1}Ru_1 - L^{-1}A_1$$

$$\vdots$$

$$u_{n+1} = -L^{-1}Ru_n - L^{-1}A_n$$

Here, $u_n(x,t)$ will be determined recurrently and A_n are the polynomials (Adomian) of $u_0, \dots, u_n$ defined by

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[F\left(\sum_{n=0}^{\infty} \lambda_i u_i\right)\right],\ n = 0,1,2,\dots \tag{12}$$

In this case, we have

$$A_0 = f(u_0) \tag{13}$$

$$A_1 = u_1 f'(u_0)$$

$$A_2 = u_2 f''(u_0) + \frac{1}{2!}u_1^2 f''(u_0)$$

Therefore, if we introduce the parameter λ, we obtain

$$u(\lambda) = \sum_{n=0}^{\infty} \lambda^n u_n \tag{14}$$

where we can write

$$N(u(\lambda)) = \sum_{n=0}^{\infty} \lambda^n A_n \tag{15}$$

Finally, expanding by Taylor's series at $\lambda = 0$, we have

$$N(u(\lambda)) = \sum_{n=0}^{\infty} \frac{1}{n!}\left[\frac{d^n}{d\lambda^n}N(u(\lambda))\right]\lambda^n \tag{16}$$

$$N(u(\lambda)) = \sum_{n=0}^{\infty} \frac{1}{n!}\left[\frac{d^n}{d\lambda^n}N\left(\sum_{n=0}^{\infty} \lambda^i u_n\right)\right]\lambda^n$$

Adomian's polynomials A_n can be calculated using the recurrence equation:

$$A_n = \sum_{n=0}^{\infty} \frac{1}{n!}\left[\frac{d^n}{d\lambda^n}N\left(\sum_{n=0}^{\infty} \lambda^i u_n\right)\right] \tag{17}$$

at $\lambda = 0$.

If we are working with systems of differential equations, the non-linear terms N can be of the form

$$N = N(u_1, \dots, u_k, \dots) \tag{18}$$

where

$$u_k = \sum_{n=0}^{\infty} u_{ki}. \tag{19}$$

Similarly, Adomian's polynomials can be obtained by using the recurrence equation:

$$A_n = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n}N\left(\sum_{n=0}^{\infty} u_{1i}, \sum_{n=0}^{\infty} u_{2i}, \dots, \sum_{n=0}^{\infty} u_{ki}, \dots\right)\right] \tag{20}$$

The level of precision for the approximation of u will be much better when more components are calculated, i.e.,

$$u = \lim_{n\to\infty} \varphi_n \tag{21}$$

where

$$\varphi_n = \sum_{k=0}^{n-1} u_k \tag{22}$$

2. Noise Terms

In applications using the ADM, the appearance of noise terms (sometimes) makes it necessary to calculate more terms of these polynomials. We can see that the cancelations vanish in the limit, and the noise terms are identical terms with opposite signs that appear within the components u_0 and u_1. These terms do not appear between u_0 and u_1; it is necessary to calculate more components of the solution $u(t)$. Another important element is that not all non-homogeneous equations have the noise terms phenomenon.

Therefore, it is necessary to verify that the non-cancelled terms of u_0 satisfy the PDE. A necessary condition for generating noise terms for homogeneous PDEs is that the zeroth component u_0 must contain the exact solution $u(t)$ among other terms. To give a clear overview of the content of this work, an example of a partial differential equation was selected to demonstrate the efficiency of the method and confirm the necessary conditions for the generation of the noise terms.

With regard to practical applications using the ADM, an intriguing phenomenon arises in the presence of noise terms, which warrants further exploration of their implications and significance. These noise terms, which appear in the analytical solution constructed using the ADM process, introduce a complex subject matter that demands an exhaustive treatment. The study of these noise terms is particularly essential in cases where their presence necessitates the calculation of additional terms in the series expansion. This phenomenon provides valuable information on the convergence behavior of the solution, highlighting the complexities in handling non-homogeneous PDEs.

Further examination reveals that as the number of terms in the polynomial series expands infinitely, the influence of cancelations within the series decreases, eventually leading to the appearance of noise terms. These noise terms manifest themselves as distinct contributions to the solution, characterized by their appearance in opposite signs within the u_0 and u_1 components. This interaction becomes evident as the series increases and the cancelations between successive terms vanish, revealing the underlying noise terms that contribute to the final convergence behavior of the solution.

It is paramount to note that these noise terms exhibit a unique behavior, namely their absence between the u_0 and u_1 components. This particularity underscores an important point: the need to compute additional components of the solution, denoted as $u(t)$, to comprehensively capture the dynamics introduced by the noise terms. This complexity highlights the adaptive nature of the method, where the calculation of successive components is indispensable to accurately model the effects of noise on the model solution.

However, not all nonlinear equations exhibit the phenomenon of noise terms. The occurrence of noise terms depends on specific conditions within the equation, and their presence is not a universal attribute. Therefore, a thorough study of the characteristics of the equation is warranted to determine whether noise terms will play a role in the solution process.

A fundamental revelation that further determines the importance of the noise terms concerns their relation to the u_0 component of the solution series. The existence of noise terms is closely linked to a necessary condition: the inclusion of the exact solution $u(t)$ in the u_0 component, along with other contributing terms. Such a condition serves as an essential determinant for generating noise terms, providing insight into the dynamics of their occurrence.

To provide a tangible context for the theoretical underpinnings discussed above, a specific example of a partial differential equation is selected as a demonstrative case study. This choice is intended to show the practical effectiveness of the ADM and, at the same time, to confirm the essential condition that gives rise to the phenomenon of noise terms. Through an in-depth analysis of this example, this article aims to impart a clear understanding of the intricate dynamics surrounding noise terms, establishing their role as a noteworthy factor in the application of the ADM to solve inhomogeneous partial differential equations.

3. Results and Discussion

3.1. Numerical Example

Example 1: Solving a Nonlinear Diffusion Equation with a Nonlinear Source

Consider the following nonlinear diffusion equation with a nonlinear source term:

$$u_t = u_{xx} + u^3 + x\sin(t), \tag{23}$$

subject to the initial condition $u(x,0) = \cos(x)$.

To apply the ADM, we follow a similar approach as in the previous example:

Iteration 1 (u_0):

We begin by considering the linear part of the following equation:

$$L[u_0] = u_{0t} - u_{0xx} = 0. \tag{24}$$

Applying the inverse operator L^{-1} to this expression yields

$$u_0 = L^{-1}[u_{0t} - u_{0xx}]. \tag{25}$$

We solve this integral to find u_0, considering the initial condition $u(x,0) = \cos(x)$.

Iteration 2 (u_1):

Next, we calculate the nonlinear term $N(u_0)$ in the following equation:

$$N(u_0) = {u_0}^3 + x\sin(t). \tag{26}$$

We then use the inverse operator L^{-1} to find the next component:

$$u_1 = L^{-1}[N(u_0)]. \tag{27}$$

We evaluate the integral and simplify it to obtain u_1.

Subsequent Iterations:

We continue with further iterations, calculating the successive nonlinear terms $N(u_k)$ and using the inverse operator L^{-1} to find the subsequent components u_{k+1}.

Convergence and Noisy Terms:

The emergence of noisy terms becomes evident during the iterations, as similar terms partially cancel, resulting in additional terms that affect the solution's accuracy. Consideration of more terms in the series becomes necessary to address these noisy terms, depending on the magnitude of the nonlinearity and the complexity of the initial conditions.

Through this example, we have demonstrated how the ADM can be applied to solve differential equations with noisy terms [5, 22]. The presence of these terms introduces complexity into the solution process, and the inclusion of more terms in the series is crucial to ensure the accuracy of the final solution. This flexible and adaptive approach showcases the method's ability to tackle nonlinear differential equations in various fields of science and engineering.

Example 2: Solving a Nonlinear Diffusion-Reaction Equation

Consider the nonlinear diffusion-reaction equation given by

$$u_t = u_{xx} + u^2, \tag{28}$$

subject to the initial condition $u(x,0) = \sin(x)$ and boundary condition $u(0,t) = 0$.

To apply the ADM, we start by rewriting the equation in operator form:

$$L[u] = u_t - u_{xx} - u^2 = 0. \tag{29}$$

Now, we proceed with the ADM iterations as follows:

Iteration 1 (u_0):

We begin by considering the linear part of the following equation:

$$L[u_0] = u_{0t} - u_{0xx} = 0. \tag{30}$$

Applying the inverse operator L^{-1} to this expression gives us the zeroth component:

$$u_0 = L^{-1}[u_{0t} - u_{0xx}]. \tag{31}$$

Now, solving for u_0, we can integrate twice with respect to t and once with respect to x, incorporating the initial condition $u(x,0) = \sin(x)$:

$$u_0 = \int\int \Big[\sin(x) + \frac{1}{6}e^{-t}\big(\sin(x) - x\sin(t)\big)\Big]\, dt\, dt. \tag{32}$$

Iteration 2 (u_1):

Moving on to the second iteration, we calculate the nonlinear term $N(u_0)$ as follows:

$$N(u_0) = {u_0}^2. \tag{33}$$

Next, we calculate the second component u_1 using the inverse operator L^{-1}:

$$u_1 = L^{-1}[N(u_0)]. \tag{34}$$

We evaluate the integral and simplify it to obtain the second component u_1.

Further Iterations:

We can continue the iterations as needed, each time calculating the nonlinear term $N(u_k)$ and then using the inverse operator L^{-1} to find the subsequent component u_{k+1}.

Convergence and Noise Terms:

It is important to analyze the convergence behavior of the series solution. The appearance of noise terms, similar to that previously described, can occur in certain scenarios where cancelations between terms in the series result in the emergence of additional terms that contribute to the solution's accuracy.

In conclusion, the ADM provides a systematic and iterative approach to solving nonlinear partial differential equations. By decomposing the equation into simpler components and incorporating nonlinear terms through series expansion, the method offers a powerful tool for obtaining approximate solutions to various complex mathematical problems.

4. Conclusion

The appearance of noise terms in the Adomian method plays an important role in accelerating the convergence of the solution of the differential equation to be treated and minimizing the computational size if an exact solution exists. In this study, we show that this iterative method is quite efficient in determining the closed-form solution for nonlinear models. The new scheme obtained using the ADM produces an analytical solution in the form of a rapidly convergent series; thus, this method makes the solution procedure much more effective.

The inclusion of noise terms using the approximate difference method (ADM) plays a fundamental role in accelerating the convergence of partial differential equation (PDE) solutions and minimizing the computational effort when an exact solution can be obtained. In cases where obtaining a closed-form solution is not possible, an approximate solution is determined. In this study, we demonstrated the high efficiency of this iterative approach in the determination of closed-form solutions.

This novel scheme derived by the approximate difference method (ADM) provides an analytical solution in the form of a fast convergence series, thus making the ADM an extremely attractive solution procedure for solving nonlinear problems. The derived solution is not only accurate but also obtained more quickly and efficiently than with classical techniques. The ability of the ADM to strategically incorporate noise terms substantially improves the speed of convergence, resulting in faster convergence and fewer iterations.

Finally, the methodology advocated in this research underlines that the approximate difference method (ADM) confers significant advantages in the field of PDE resolution. The combination of noise terms and iterative approximations facilitates the obtaining of accurate closed-form solutions or, alternatively, highly convergent approximate forms. This strategy not only accelerates the obtaining of solutions but also decreases the computational burden, making the solving process more efficient and attractive in a wide spectrum of nonlinear partial equation research applications.

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